Southfields Academy

Mathematics department

A-level mathematics entrance exam revision booklet

Any questions: Please email Mr. Alex Spencer

alexander.spencer@southfieldsacademy.com

Southfields Academy A-Level Mathematics Entrance Exam Revision booklet

- This booklet is for students who got a grade 6 in the GCSE mathematics exam.
- In order to be accepted on to the mathematics A-level course you will need to pass a challenging exam which will take place at the beginning of September.
- The topics tested in this exam will be:
  - Indices and surds
  - Factorising
  - Solving algebraic equations
  - Changing the subject of a formula
  - Coordinate geometry

- Please read the following pages to help prepare you and try the questions. The answers are at the back of the document.

- If you have any questions please email Mr. Alex Spencer

alexander.spencer@southfieldsacademy.com

1.1 Index laws

- **You can use the laws of indices to simplify powers of the same base.**
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^n = a^n b^n$

Notation

x^5 — This is the **base**. This is the **index**, **power** or **exponent**.

Example 1

Simplify these expressions:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $\frac{b^7}{b^4}$ **d** $6x^5 \div 3x^3$ **e** $(a^3)^2 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

a $x^2 \times x^5 = x^{2+5} = x^7$

Use the rule $a^m \times a^n = a^{m+n}$ to simplify the index.

b $2r^2 \times 3r^3 = 2 \times 3 \times r^2 \times r^3$
$= 6 \times r^{2+3} = 6r^5$

Rewrite the expression with the numbers together and the r terms together.
$2 \times 3 = 6$
$r^2 \times r^3 = r^{2+3}$

c $\frac{b^7}{b^4} = b^{7-4} = b^3$

Use the rule $a^m \div a^n = a^{m-n}$ to simplify the index.

d $6x^5 \div 3x^3 = \frac{6}{3} \times \frac{x^5}{x^3}$
$= 2 \times x^2 = 2x^2$

$x^5 \div x^3 = x^{5-3} = x^2$

e $(a^3)^2 \times 2a^2 = a^6 \times 2a^2$
$= 2 \times a^6 \times a^2 = 2a^8$

Use the rule $(a^m)^n = a^{mn}$ to simplify the index.

$a^6 \times a^2 = a^{6+2} = a^8$

f $\frac{(3x^2)^3}{x^4} = 3^3 \times \frac{(x^2)^3}{x^4}$
$= 27 \times \frac{x^6}{x^4} = 27x^2$

Use the rule $(ab)^n = a^n b^n$ to simplify the numerator.
$(x^2)^3 = x^{2 \times 3} = x^6$

$\frac{x^6}{x^4} = x^{6-4} = x^2$

Exercise 1A

1 Simplify these expressions:

a $x^3 \times x^4$ **b** $2x^3 \times 3x^2$ **c** $\frac{k^3}{k^2}$

d $\frac{4p^3}{2p}$ **e** $\frac{3x^3}{3x^2}$ **f** $(y^2)^5$

g $10x^5 \div 2x^3$ **h** $(p^3)^2 \div p^4$ **i** $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$ **k** $2a^4 \times 3a^5$ **l** $\frac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$ **n** $3x^3 \times 2x^2 \times 4x^6$ **o** $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$ **q** $2a^3 \div 3a^2 \times 6a^5$ **r** $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x - 2)$ **b** $x(x + 9)$ **c** $-3y(4 - 3y)$

d $x(y + 5)$ **e** $-x(3x + 5)$ **f** $-5x(4x + 1)$

g $(4x + 5)x$ **h** $-3y(5 - 2y^2)$ **i** $-2x(5x - 4)$

j $(3x - 5)x^2$ **k** $3(x + 2) + (x - 7)$ **l** $5x - 6 - (3x - 2)$

m $4(c + 3d^2) - 3(2c + d^2)$ **n** $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

o $x(3x^2 - 2x + 5)$ **p** $7y^2(2 - 5y + 3y^2)$ **q** $-2y^2(5 - 7y + 3y^2)$

r $7(x - 2) + 3(x + 4) - 6(x - 2)$ **s** $5x - 3(4 - 2x) + 6$

t $3x^2 - x(3 - 4x) + 7$ **u** $4x(x + 3) - 2x(3x - 7)$ **v** $3x^2(2x + 1) - 5x^2(3x - 4)$

3 Simplify these fractions:

a $\dfrac{6x^4 + 10x^6}{2x}$ **b** $\dfrac{3x^5 - x^7}{x}$ **c** $\dfrac{2x^4 - 4x^2}{4x}$

d $\dfrac{8x^3 + 5x}{2x}$ **e** $\dfrac{7x^7 + 5x^2}{5x}$ **f** $\dfrac{9x^5 - 5x^3}{3x}$

Example 5

Expand these expressions and simplify if possible:

a $x(2x + 3)(x - 7)$ **b** $x(5x - 3y)(2x - y + 4)$ **c** $(x - 4)(x + 3)(x + 1)$

a $x(2x + 3)(x - 7)$

$= (2x^2 + 3x)(x - 7)$

$= 2x^3 - 14x^2 + 3x^2 - 21x$

$= 2x^3 - 11x^2 - 21x$

Start by expanding one pair of brackets: $x(2x + 3) = 2x^2 + 3x$

You could also have expanded the second pair of brackets first: $(2x + 3)(x - 7) = 2x^2 - 11x - 21$ Then multiply by x.

b $x(5x - 3y)(2x - y + 4)$

$= (5x^2 - 3xy)(2x - y + 4)$

$= 5x^2(2x - y + 4) - 3xy(2x - y + 4)$

$= 10x^3 - 5x^2y + 20x^2 - 6x^2y + 3xy^2 - 12xy$

$= 10x^3 - 11x^2y + 20x^2 + 3xy^2 - 12xy$

Be careful with minus signs. You need to change every sign in the second pair of brackets when you multiply it out.

c $(x - 4)(x + 3)(x + 1)$

$= (x^2 - x - 12)(x + 1)$

$= x^2(x + 1) - x(x + 1) - 12(x + 1)$

$= x^3 + x^2 - x^2 - x - 12x - 12$

$= x^3 - 13x - 12$

Choose one pair of brackets to expand first, for example:

$$(x - 4)(x + 3) = x^2 + 3x - 4x - 12$$
$$= x^2 - x - 12$$

You multiplied together three linear terms, so the final answer contains an x^3 term.

Exercise 1B

1 Expand and simplify if possible:

a $(x+4)(x+7)$ **b** $(x-3)(x+2)$ **c** $(x-2)^2$

d $(x-y)(2x+3)$ **e** $(x+3y)(4x-y)$ **f** $(2x-4y)(3x+y)$

g $(2x-3)(x-4)$ **h** $(3x+2y)^2$ **i** $(2x+8y)(2x+3)$

j $(x+5)(2x+3y-5)$ **k** $(x-1)(3x-4y-5)$ **l** $(x-4y)(2x+y+5)$

m $(x+2y-1)(x+3)$ **n** $(2x+2y+3)(x+6)$ **o** $(4-y)(4y-x+3)$

p $(4y+5)(3x-y+2)$ **q** $(5y-2x+3)(x-4)$ **r** $(4y-x-2)(5-y)$

2 Expand and simplify if possible:

a $5(x+1)(x-4)$ **b** $7(x-2)(2x+5)$ **c** $3(x-3)(x-3)$

d $x(x-y)(x+y)$ **e** $x(2x+y)(3x+4)$ **f** $y(x-5)(x+1)$

g $y(3x-2y)(4x+2)$ **h** $y(7-x)(2x-5)$ **i** $x(2x+y)(5x-2)$

j $x(x+2)(x+3y-4)$ **k** $y(2x+y-1)(x+5)$ **l** $y(3x+2y-3)(2x+1)$

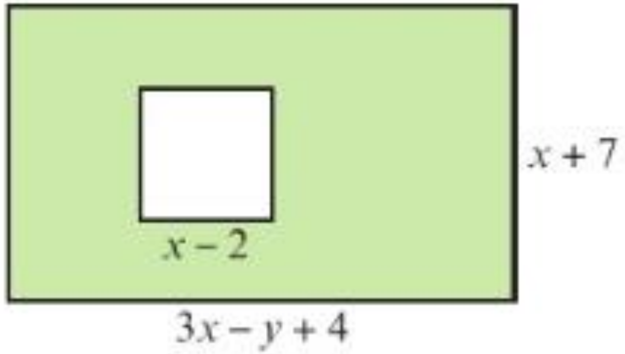
m $x(2x+3)(x+y-5)$ **n** $2x(3x-1)(4x-y-3)$ **o** $3x(x-2y)(2x+3y+5)$

p $(x+3)(x+2)(x+1)$ **q** $(x+2)(x-4)(x+3)$ **r** $(x+3)(x-1)(x-5)$

s $(x-5)(x-4)(x-3)$ **t** $(2x+1)(x-2)(x+1)$ **u** $(2x+3)(3x-1)(x+2)$

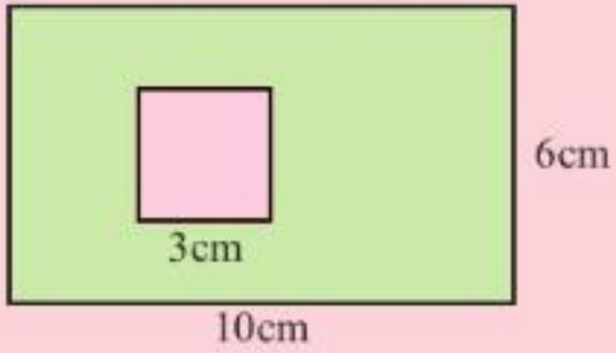
v $(3x-2)(2x+1)(3x-2)$ **w** $(x+y)(x-y)(x-1)$ **x** $(2x-3y)^3$

3 The diagram shows a rectangle with a square cut out.
The rectangle has length $3x - y + 4$ and width $x + 7$.
The square has length $x - 2$.
Find an expanded and simplified expression for the shaded area.



Problem-solving

Use the same strategy as you would use if the lengths were given as numbers:



4 A cuboid has dimensions $x + 2$ cm, $2x - 1$ cm and $2x + 3$ cm.
Show that the volume of the cuboid is $4x^3 + 12x^2 + 5x - 6$ cm^3.

5 Given that $(2x + 5y)(3x - y)(2x + y) = ax^3 + bx^2y + cxy^2 + dy^3$, where a, b, c and d are constants, find the values of a, b, c and d. **(2 marks)**

Challenge

Expand and simplify $(x + y)^4$.

Links You can use the binomial expansion to expand expressions like $(x + y)^4$ quickly. → Section 8.3

Example 7

- **A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.**

Factorise:

a $x^2 - 5x - 6$ **b** $x^2 + 6x + 8$ **c** $6x^2 - 11x - 10$ **d** $x^2 - 25$ **e** $4x^2 - 9y^2$

a $x^2 - 5x - 6$

$ac = -6$ and $b = -5$

So $x^2 - 5x - 6 = x^2 + x - 6x - 6$

$= x(x + 1) - 6(x + 1)$

$= (x + 1)(x - 6)$

b $x^2 + 6x + 8$

$= x^2 + 2x + 4x + 8$

$= x(x + 2) + 4(x + 2)$

$= (x + 2)(x + 4)$

c $6x^2 - 11x - 10$

$= 6x^2 - 15x + 4x - 10$

$= 3x(2x - 5) + 2(2x - 5)$

$= (2x - 5)(3x + 2)$

d $x^2 - 25$

$= x^2 - 5^2$

$= (x + 5)(x - 5)$

e $4x^2 - 9y^2$

$= 2^2x^2 - 3^2y^2$

$= (2x + 3y)(2x - 3y)$

Exercise 1C

1 Factorise these expressions completely:

1 Factorise these expressions completely:

a $4x + 8$ **b** $6x - 24$ **c** $20x + 15$

d $2x^2 + 4$ **e** $4x^2 + 20$ **f** $6x^2 - 18x$

g $x^2 - 7x$ **h** $2x^2 + 4x$ **i** $3x^2 - x$

j $6x^2 - 2x$ **k** $10y^2 - 5y$ **l** $35x^2 - 28x$

m $x^2 + 2x$ **n** $3y^2 + 2y$ **o** $4x^2 + 12x$

p $5y^2 - 20y$ **q** $9xy^2 + 12x^2y$ **r** $6ab - 2ab^2$

s $5x^2 - 25xy$ **t** $12x^2y + 8xy^2$ **u** $15y - 20yz^2$

v $12x^2 - 30$ **w** $xy^2 - x^2y$ **x** $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$ **b** $2x^2 + 6x$ **c** $x^2 + 11x + 24$

d $x^2 + 8x + 12$ **e** $x^2 + 3x - 40$ **f** $x^2 - 8x + 12$

g $x^2 + 5x + 6$ **h** $x^2 - 2x - 24$ **i** $x^2 - 3x - 10$

j $x^2 + x - 20$ **k** $2x^2 + 5x + 2$ **l** $3x^2 + 10x - 8$

m $5x^2 - 16x + 3$ **n** $6x^2 - 8x - 8$

o $2x^2 + 7x - 15$ **p** $2x^4 + 14x^2 + 24$

Hint For part **n**, take 2 c
factor first. For part **p**, le

q $x^2 - 4$ **r** $x^2 - 49$

s $4x^2 - 25$ **t** $9x^2 - 25y^2$ **u** $36x^2 - 4$

v $2x^2 - 50$ **w** $6x^2 - 10x + 4$ **x** $15x^2 + 42x - 9$

3 Factorise completely:

a $x^3 + 2x$
b $x^3 - x^2 + x$
c $x^3 - 5x$
d $x^3 - 9x$
e $x^3 - x^2 - 12x$
f $x^3 + 11x^2 + 30x$
g $x^3 - 7x^2 + 6x$
h $x^3 - 64x$
i $2x^3 - 5x^2 - 3x$
j $2x^3 + 13x^2 + 15x$
k $x^3 - 4x$
l $3x^3 + 27x^2 + 60x$

4 Factorise completely $x^4 - y^4$. **(2 marks)**

Problem-solving

Watch out for terms that can be written as a function of a function: $x^4 = (x^2)^2$

5 Factorise completely $6x^3 + 7x^2 - 5x$. **(2 marks)**

Challenge

Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

1.4 Negative and fractional indices

- **You can use the laws of indices with any rational power.**
  - $a^{\frac{1}{m}} = \sqrt[m]{a}$
  - $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
  - $a^{-m} = \frac{1}{a^m}$
  - $a^0 = 1$

Example 9

Simplify:

a $\dfrac{x^3}{x^{-3}}$ **b** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c** $(x^3)^{\frac{2}{3}}$ **d** $2x^{1.5} \div 4x^{-0.25}$ **e** $\sqrt[3]{125x^6}$ **f** $\dfrac{2x^2 - x}{x^5}$

a $\dfrac{x^3}{x^{-3}} = x^{3-(-3)} = x^6$

Use the rule $a^m \div a^n = a^{m-n}$.

b $x^{\frac{1}{2}} \times x^{\frac{3}{2}} = x^{\frac{1}{2}+\frac{3}{2}} = x^2$

This could also be written as $\sqrt{x}$.

Use the rule $a^m \times a^n = a^{m+n}$.

c $(x^3)^{\frac{2}{3}} = x^{3 \times \frac{2}{3}} = x^2$

Use the rule $(a^m)^n = a^{mn}$.

d $2x^{1.5} \div 4x^{-0.25} = \frac{1}{2}x^{1.5-(-0.25)} = \frac{1}{2}x^{1.75}$

Use the rule $a^m \div a^n = a^{m-n}$.
$1.5 - (-0.25) = 1.75$

e $\sqrt[3]{125x^6} = (125x^6)^{\frac{1}{3}}$

$= (125)^{\frac{1}{3}}(x^6)^{\frac{1}{3}} = \sqrt[3]{125}(x^{6 \times \frac{1}{3}}) = 5x^2$

Using $a^{\frac{1}{m}} = \sqrt[m]{a}$.

f $\dfrac{2x^2 - x}{x^5} = \dfrac{2x^2}{x^5} - \dfrac{x}{x^5}$

Divide each term of the numerator by x^5.

$= 2 \times x^{2-5} - x^{1-5} = 2x^{-3} - x^{-4}$

$= \dfrac{2}{x^3} - \dfrac{1}{x^4}$

Using $a^{-m} = \dfrac{1}{a^m}$

Example 11

Given that $y = \frac{1}{16}x^2$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{2}}$

b $4y^{-1}$

a $y^{\frac{1}{2}} = \left(\frac{1}{16}x^2\right)^{\frac{1}{2}}$

$= \frac{1}{\sqrt{16}}x^{2 \times \frac{1}{2}} = \frac{x}{4}$

b $4y^{-1} = 4\left(\frac{1}{16}x^2\right)^{-1}$

$= 4\left(\frac{1}{16}\right)^{-1}x^{2 \times (-1)} = 4 \times 16x^{-2}$

$= 64x^{-2}$

Substitute $y = \frac{1}{16}x^2$ into $y^{\frac{1}{2}}$.

$\left(\frac{1}{16}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{16}}$ and $(x^2)^{\frac{1}{2}} = x^{2 \times \frac{1}{2}}$

$\left(\frac{1}{16}\right)^{-1} = 16$ and $x^{2 \times -1} = x^{-2}$

Problem-solving

Check that your answers are in the correct form. If k and n are constants they could be positive or negative, and they could be integers, fractions or surds.

Exercise 1D

surds.

Exercise 1D

1 Simplify:

a $x^3 \div x^{-2}$

b $x^5 \div x^7$

c $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$

e $(x^3)^{\frac{5}{3}}$

f $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$

h $5x^{\frac{7}{5}} \div x^{\frac{2}{5}}$

i $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$

k $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$

l $\dfrac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$

b $81^{\frac{3}{2}}$

c $27^{\frac{1}{3}}$

d 4^{-2}

e $9^{-\frac{1}{2}}$

f $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$

h $1296^{\frac{3}{4}}$

i $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

k $\left(\frac{6}{5}\right)^{-1}$

l $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\dfrac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\dfrac{x + 4x^3}{x^3}$

e $\dfrac{2x + x^2}{x^4}$ **f** $\left(\dfrac{4}{9}x^4\right)^{\frac{3}{2}}$ **g** $\dfrac{9x^2 - 15x^5}{3x^3}$ **h** $\dfrac{5x + 3x^2}{15x^3}$

4 **a** Find the value of $81^{\frac{1}{4}}$. **(1 mark)**

b Simplify $x(2x^{-\frac{1}{3}})^4$. **(2 marks)**

5 Given that $y = \frac{1}{8}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(2 marks)**

b $\frac{1}{2}y^{-2}$ **(2 marks)**

1.5 Surds

If n is an integer that is **not** a square number, then any multiple of $\sqrt{n}$ is called a surd.
Examples of surds are $\sqrt{2}$, $\sqrt{19}$ and $5\sqrt{2}$.

Notation Irrational numbers cannot be written in the form $\frac{a}{b}$ where a and b are integers.
Surds are examples of **irrational numbers**.

Surds are examples of **irrational numbers**.
The decimal expansion of a surd is never-ending and never repeats, for example $\sqrt{2} = 1.414213562\ldots$

You can use surds to write exact answers to calculations.

- **You can manipulate surds using these rules:**
  - $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
  - $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example 13

Expand and simplify if possible:

a $\sqrt{2}(5 - \sqrt{3})$ **b** $(2 - \sqrt{3})(5 + \sqrt{3})$

a $\sqrt{2}(5 - \sqrt{3})$

$= 5\sqrt{2} - \sqrt{2}\sqrt{3}$

$= 5\sqrt{2} - \sqrt{6}$

b $(2 - \sqrt{3})(5 + \sqrt{3})$

$= 2(5 + \sqrt{3}) - \sqrt{3}(5 + \sqrt{3})$

$= 10 + 2\sqrt{3} - 5\sqrt{3} - \sqrt{9}$

$= 7 - 3\sqrt{3}$

Exercise 1E

1 Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$ **b** $\sqrt{72}$ **c** $\sqrt{50}$

d $\sqrt{32}$ **e** $\sqrt{90}$ **f** $\frac{\sqrt{12}}{2}$

g $\frac{\sqrt{27}}{3}$ **h** $\sqrt{20}+\sqrt{80}$ **i** $\sqrt{200}+\sqrt{18}-\sqrt{72}$

j $\sqrt{175}+\sqrt{63}+2\sqrt{28}$ **k** $\sqrt{28}-2\sqrt{63}+\sqrt{7}$ **l** $\sqrt{80}-2\sqrt{20}+3\sqrt{45}$

m $3\sqrt{80}-2\sqrt{20}+5\sqrt{45}$ **n** $\frac{\sqrt{44}}{\sqrt{11}}$ **o** $\sqrt{12}+3\sqrt{48}+\sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2+\sqrt{3})$ **b** $\sqrt{5}(3-\sqrt{3})$ **c** $\sqrt{2}(4-\sqrt{5})$

d $(2-\sqrt{2})(3+\sqrt{5})$ **e** $(2-\sqrt{3})(3-\sqrt{7})$ **f** $(4+\sqrt{5})(2+\sqrt{5})$

g $(5-\sqrt{3})(1-\sqrt{3})$ **h** $(4+\sqrt{3})(2-\sqrt{3})$ **i** $(7-\sqrt{11})(2+\sqrt{11})$

3 Simplify $\sqrt{75}-\sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer.

1.6 Rationalising denominators

If a fraction has a surd in the denominator, it is sometimes useful to **rearrange** it so that the denominator is a **rational** number. This is called rationalising the denominator.

- **The rules to rationalise denominators are:**
  - **For fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.**
  - **For fractions in the form $\frac{1}{a+\sqrt{b}}$, multiply the numerator and denominator by $a-\sqrt{b}$.**
  - **For fractions in the form $\frac{1}{a-\sqrt{b}}$, multiply the numerator and denominator by $a+\sqrt{b}$.**

Example 14

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{3+\sqrt{2}}$ **c** $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ **d** $\frac{1}{(1-\sqrt{3})^2}$

a $\dfrac{1}{\sqrt{3}} = \dfrac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$ — Multiply the numerator and denominator by $\sqrt{3}$.

$= \dfrac{\sqrt{3}}{3}$ — $\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$

b $\dfrac{1}{3 + \sqrt{2}} = \dfrac{1 \times (3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$ — Multiply numerator and denominator by $(3 - \sqrt{2})$.

$= \dfrac{3 - \sqrt{2}}{9 - 3\sqrt{2} + 3\sqrt{2} - 2}$ — $\sqrt{2} \times \sqrt{2} = 2$

$= \dfrac{3 - \sqrt{2}}{7}$ — $9 - 2 = 7$, $-3\sqrt{2} + 3\sqrt{2} = 0$

c $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \dfrac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$

Multiply numerator and denominator by $\sqrt{5}+\sqrt{2}$.

$= \dfrac{5+\sqrt{5}\sqrt{2}+\sqrt{2}\sqrt{5}+2}{5-2}$

$-\sqrt{2}\sqrt{5}$ and $\sqrt{5}\sqrt{2}$ cancel each other out.

$= \dfrac{7+2\sqrt{10}}{3}$

$\sqrt{5}\sqrt{2} = \sqrt{10}$

d $\dfrac{1}{(1-\sqrt{3})^2} = \dfrac{1}{(1-\sqrt{3})(1-\sqrt{3})}$

Expand the brackets.

$= \dfrac{1}{1-\sqrt{3}-\sqrt{3}+\sqrt{9}}$

Simplify and collect like terms. $\sqrt{9}=3$

$= \dfrac{1}{4-2\sqrt{3}}$

$= \dfrac{1\times(4+2\sqrt{3})}{(4-2\sqrt{3})(4+2\sqrt{3})}$

Multiply the numerator and denominator by $4+2\sqrt{3}$.

$\sqrt{3}\times\sqrt{3}=3$

$= \dfrac{4+2\sqrt{3}}{16+8\sqrt{3}-8\sqrt{3}-12}$

$16-12=4$, $8\sqrt{3}-8\sqrt{3}=0$

$= \dfrac{4+2\sqrt{3}}{4} = \dfrac{2+\sqrt{3}}{2}$

Exercise 1F

1 Simplify:

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$ **c** $\frac{1}{\sqrt{2}}$ **d** $\frac{\sqrt{3}}{\sqrt{15}}$

e $\frac{\sqrt{12}}{\sqrt{48}}$ **f** $\frac{\sqrt{5}}{\sqrt{80}}$ **g** $\frac{\sqrt{12}}{\sqrt{156}}$ **h** $\frac{\sqrt{7}}{\sqrt{63}}$

2 Rationalise the denominators and simplify:

a $\frac{1}{1+\sqrt{3}}$ **b** $\frac{1}{2+\sqrt{5}}$ **c** $\frac{1}{3-\sqrt{7}}$ **d** $\frac{4}{3-\sqrt{5}}$ **e** $\frac{1}{\sqrt{5}-\sqrt{3}}$

f $\frac{3-\sqrt{2}}{4-\sqrt{5}}$ **g** $\frac{5}{2+\sqrt{5}}$ **h** $\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ **i** $\frac{11}{3+\sqrt{11}}$ **j** $\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

k $\frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$ **l** $\frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$ **m** $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

3 Rationalise the denominators and simplify:

a $\dfrac{1}{(3-\sqrt{2})^2}$ **b** $\dfrac{1}{(2+\sqrt{5})^2}$ **c** $\dfrac{4}{(3-\sqrt{2})^2}$

d $\dfrac{3}{(5+\sqrt{2})^2}$ **e** $\dfrac{1}{(5+\sqrt{2})(3-\sqrt{2})}$ **f** $\dfrac{2}{(5-\sqrt{3})(2+\sqrt{3})}$

4 Simplify $\dfrac{3-2\sqrt{5}}{\sqrt{5}-1}$ giving your answer in the form $p+q\sqrt{5}$, where p and q are rational numbers. **(4 marks)**

Problem-solving

You can check that your answer is in the correct form by writing down the values of p and q and checking that they are rational numbers.

Mixed exercise 1

1 Simplify:

a $y^3 \times y^5$ **b** $3x^2 \times 2x^5$ **c** $(4x^2)^3 \div 2x^5$ **d** $4b^2 \times 3b^3 \times b^4$

2 Expand and simplify if possible:

a $(x + 3)(x - 5)$ **b** $(2x - 7)(3x + 1)$ **c** $(2x + 5)(3x - y + 2)$

3 Expand and simplify if possible:

a $x(x + 4)(x - 1)$ **b** $(x + 2)(x - 3)(x + 7)$ **c** $(2x + 3)(x - 2)(3x - 1)$

4 Expand the brackets:

a $3(5y + 4)$ **b** $5x^2(3 - 5x + 2x^2)$ **c** $5x(2x + 3) - 2x(1 - 3x)$ **d** $3x^2(1 + 3x) - 2x(3x - 2)$

5 Factorise these expressions completely:

a $3x^2 + 4x$ **b** $4y^2 + 10y$ **c** $x^2 + xy + xy^2$ **d** $8xy^2 + 10x^2y$

6 Factorise:

a $x^2 + 3x + 2$ **b** $3x^2 + 6x$ **c** $x^2 - 2x - 35$ **d** $2x^2 - x - 3$

e $5x^2 - 13x - 6$ **f** $6 - 5x - x^2$

7 Factorise:

a $2x^3 + 6x$ **b** $x^3 - 36x$ **c** $2x^3 + 7x^2 - 15x$

8 Simplify:

a $9x^3 \div 3x^{-3}$ **b** $(4^{\frac{3}{2}})^{\frac{1}{3}}$ **c** $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

9 Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **b** $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify:

a $\frac{3}{\sqrt{63}}$ **b** $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

11 **a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.

b By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.

12 Expand and simplify if possible:

a $\sqrt{2}(3 + \sqrt{5})$ **b** $(2 - \sqrt{5})(5 + \sqrt{3})$ **c** $(6 - \sqrt{2})(4 - \sqrt{7})$

13 Rationalise the denominator and simplify:

a $\dfrac{1}{\sqrt{3}}$ **b** $\dfrac{1}{\sqrt{2} - 1}$ **c** $\dfrac{3}{\sqrt{3} - 2}$ **d** $\dfrac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$ **e** $\dfrac{1}{(2 + \sqrt{3})^2}$ **f** $\dfrac{1}{(4 - \sqrt{7})^2}$

14 **a** Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.

b Hence, fully factorise $x^3 - x^2 - 17x - 15$.

15 Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(1 mark)**

b $4y^{-1}$ **(1 mark)**

16 Show that $\dfrac{5}{\sqrt{75}-\sqrt{50}}$ can be written in the form $\sqrt{a}+\sqrt{b}$, where a and b are integers. **(5 marks)**

17 Expand and simplify $(\sqrt{11}-5)(5-\sqrt{11})$. **(2 marks)**

18 Factorise completely $x-64x^3$. **(3 marks)**

19 Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

20 Solve the equation $8 + x\sqrt{12} = \dfrac{8x}{\sqrt{3}}$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. **(4 marks)**

21 A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm^2.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

22 Show that $\dfrac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

23 Given that $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

24 Given that $\dfrac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a and the value of b. **(2 marks)**

Challenge

a Simplify $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$.

b Hence show that $\dfrac{1}{\sqrt{1} + \sqrt{2}} + \dfrac{1}{\sqrt{2} + \sqrt{3}} + \dfrac{1}{\sqrt{3} + \sqrt{4}} + \ldots + \dfrac{1}{\sqrt{24} + \sqrt{25}} = 4$

Summary of key points

1 You can use the laws of indices to simplify powers of the **same base**.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$

2 Factorising is the opposite of expanding brackets.

3 A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.

4 $x^2 - y^2 = (x + y)(x - y)$

5 You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^{-m} = \dfrac{1}{a^m}$
- $a^0 = 1$

6 You can manipulate surds using these rules:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

7 The rules to rationalise denominators are:

- Fractions in the form $\dfrac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
- Fractions in the form $\dfrac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $a - \sqrt{b}$.
- Fractions in the form $\dfrac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.

Answers

CHAPTER 1

Prior knowledge check

1 a $2m^2n + 3mn^2$ b $6x^2 - 12x - 10$
2 a 2^8 b 2^4 c 2^6
3 a $3x + 12$ b $10 - 15x$ c $12x - 30y$
4 a 8 b $2x$ c xy
5 a $2x$ b $10x$ c $\frac{5x}{3}$

Exercise 1A

1 a x^7 b $6x^5$ c k d $2p^2$
e x f y^{10} g $5x^2$ h p^2
i $2a^3$ j $2p$ k $6a^9$ l $3a^2b^3$
m $27x^8$ n $24x^{11}$ o $63a^{12}$ p $32y^6$
q $4a^6$ r $6a^{12}$

2 a $9x - 18$ b $x^2 + 9x$
c $-12y + 9y^2$ d $xy + 5x$
e $-3x^2 - 5x$ f $-20x^2 - 5x$
g $4x^2 + 5x$ h $-15y + 6y^3$
i $-10x^2 + 8x$ j $3x^3 - 5x^2$
k $4x - 1$ l $2x - 4$
m $9d^2 - 2c$ n $13 - r^2$
o $3x^3 - 2x^2 + 5x$ p $14y^2 - 35y^3 + 21y^4$
q $-10y^2 + 14y^3 - 6y^4$ r $4x + 10$
s $11x - 6$ t $7x^2 - 3x + 7$
u $-2x^2 + 26x$ v $-9x^3 + 23x^2$

3 a $3x^3 + 5x^5$ b $3x^4 - x^6$ c $\frac{x^3}{2} - x$
d $4x^2 + \frac{5}{2}$ e $\frac{7x^6}{5} + x$ f $3x^4 - \frac{5x^2}{3}$

Exercise 1B

1 a $x^2 + 11x + 28$
 b $x^2 - x - 6$
 c $x^2 - 4x + 4$
 d $2x^2 + 3x - 2xy - 3y$
 e $4x^2 + 11xy - 3y^2$
 f $6x^2 - 10xy - 4y^2$
 g $2x^2 - 11x + 12$
 h $9x^2 + 12xy + 4y^2$
 i $4x^2 + 6x + 16xy + 24y$
 j $2x^2 + 3xy + 5x + 15y - 2$
 k $3x^2 - 4xy - 8x + 4y + 5$
 l $2x^2 + 5x - 7xy - 4y^2 - 20y$
 m $x^2 + 2x + 2xy + 6y - 3$
 n $2x^2 + 15x + 2xy + 12y + 18$
 o $13y - 4x + 12 - 4y^2 + xy$
 p $12xy - 4y^2 + 3y + 15x + 10$
 q $5xy - 20y - 2x^2 + 11x - 12$
 r $22y - 4y^2 - 5x + xy - 10$

2 a $5x^2 - 15x - 20$
 b $14x^2 + 7x - 70$
 c $3x^2 - 18x + 27$
 d $x^3 - xy^2$
 e $6x^3 + 8x^2 + 3x^2y + 4xy$
 f $x^2y - 4xy - 5y$
 g $12x^2y + 6xy - 8xy^2 - 4y^2$
 h $19xy - 35y - 2x^2y$
 i $10x^3 - 4x^2 + 5x^2y - 2xy$
 j $x^3 + 3x^2y - 2x^2 + 6xy - 8x$
 k $2x^2y + 9xy + xy^2 + 5y^2 - 5y$
 l $6x^2y + 4xy^2 + 2y^2 - 3xy - 3y$
 m $2x^3 + 2x^2y - 7x^2 + 3xy - 15x$
 n $24x^3 - 6x^2y - 26x^2 + 2xy + 6x$
 o $6x^3 + 15x^2 - 3x^2y - 18xy^2 - 30xy$
 p $x^3 + 6x^2 + 11x + 6$
 q $x^3 + x^2 - 14x - 24$
 r $x^3 - 3x^2 - 13x + 15$
 s $x^3 - 12x^2 + 47x - 60$
 t $2x^3 - x^2 - 5x - 2$
 u $6x^3 + 19x^2 + 11x - 6$
 v $18x^3 - 15x^2 - 4x + 4$
 w $x^3 - xy^2 - x^2 + y^2$
 x $8x^3 - 36x^2y + 54xy^2 - 27y^3$

3 $2x^2 - xy + 29x - 7y + 24$

4 $4x^3 + 12x^2 + 5x - 6\ \text{cm}^3$

5 $a = 12, b = 32, c = 3, d = -5$

Challenge

$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Answers

Exercise 1C

1 **a** $4(x+2)$ **b** $6(x-4)$
c $5(4x+3)$ **d** $2(x^2+2)$
e $4(x^2+5)$ **f** $6x(x-3)$
g $x(x-7)$ **h** $2x(x+2)$
i $x(3x-1)$ **j** $2x(3x-1)$
k $5y(2y-1)$ **l** $7x(5x-4)$
m $x(x+2)$ **n** $y(3y+2)$
o $4x(x+3)$ **p** $5y(y-4)$
q $3xy(3y+4x)$ **r** $2ab(3-b)$
s $5x(x-5y)$ **t** $4xy(3x+2y)$
u $5y(3-4z^2)$ **v** $6(2x^2-5)$
w $xy(y-x)$ **x** $4y(3y-x)$

2 **a** $x(x+4)$ **b** $2x(x+3)$
c $(x+8)(x+3)$ **d** $(x+6)(x+2)$
e $(x+8)(x-5)$ **f** $(x-6)(x-2)$
g $(x+2)(x+3)$ **h** $(x-6)(x+4)$
i $(x-5)(x+2)$ **j** $(x+5)(x-4)$
k $(2x+1)(x+2)$ **l** $(3x-2)(x+4)$
m $(5x-1)(x-3)$ **n** $2(3x+2)(x-2)$
o $(2x-3)(x+5)$ **p** $2(x^2+3)(x^2+4)$
q $(x+2)(x-2)$ **r** $(x+7)(x-7)$
s $(2x+5)(2x-5)$ **t** $(3x+5y)(3x-5y)$
u $4(3x+1)(3x-1)$ **v** $2(x+5)(x-5)$
w $2(3x-2)(x-1)$ **x** $3(5x-1)(x+3)$

3 **a** $x(x^2+2)$ **b** $x(x^2-x+1)$
c $x(x^2-5)$ **d** $x(x+3)(x-3)$
e $x(x-4)(x+3)$ **f** $x(x+5)(x+6)$
g $x(x-1)(x-6)$ **h** $x(x+8)(x-8)$
i $x(2x+1)(x-3)$ **j** $x(2x+3)(x+5)$
k $x(x+2)(x-2)$ **l** $3x(x+4)(x+5)$

4 $(x^2+y^2)(x+y)(x-y)$

5 $x(3x+5)(2x-1)$

Challenge

$(x-1)(x+1)(2x+3)(2x-3)$

Exercise 1D

1 **a** x^5 **b** x^{-2} **c** x^4 **d** x^3
e x^5 **f** $12x^0 = 12$ **g** $3x^{\frac{1}{2}}$ **h** $5x$
i $6x^{-1}$ **j** $x^{\frac{3}{4}}$ **k** $x^{\frac{17}{6}}$ **l** $x^{\frac{1}{4}}$

2 a 5 b 729 c 3 d $\frac{1}{16}$
e $\frac{1}{3}$ f $\frac{-1}{125}$ g 1 h 216
i $\frac{125}{64}$ j $\frac{9}{4}$ k $\frac{5}{6}$ l $\frac{64}{49}$

3 a $8x^5$ b $\frac{5}{x^2} - \frac{2}{x^3}$ c $5x^4$
d $\frac{1}{x^2} + 4$ e $\frac{2}{x^3} + \frac{1}{x^2}$ f $\frac{8}{27}x^6$
g $\frac{3}{x} - 5x^2$ h $\frac{1}{3x^2} + \frac{1}{5x}$

4 a 3 b $\frac{16}{\sqrt[3]{x}}$

5 a $\frac{x}{2}$ b $\frac{32}{x^6}$

Exercise 1E

1 a $2\sqrt{7}$ b $6\sqrt{2}$ c $5\sqrt{2}$ d $4\sqrt{2}$
e $3\sqrt{10}$ f $\sqrt{3}$ g $\sqrt{3}$ h $6\sqrt{5}$
i $7\sqrt{2}$ j $12\sqrt{7}$ k $-3\sqrt{7}$ l $9\sqrt{5}$
m $23\sqrt{5}$ n 2 o $19\sqrt{3}$

2 a $2\sqrt{3} + 3$ b $3\sqrt{5} - \sqrt{15}$
c $4\sqrt{2} - \sqrt{10}$ d $6 + 2\sqrt{5} - 3\sqrt{2} - \sqrt{10}$
e $6 - 2\sqrt{7} - 3\sqrt{3} + \sqrt{21}$ f $13 + 6\sqrt{5}$
g $8 - 6\sqrt{3}$ h $5 - 2\sqrt{3}$
i $3 + 5\sqrt{11}$

3 $3\sqrt{3}$

Exercise 1F

1 a $\frac{\sqrt{5}}{5}$ b $\frac{\sqrt{11}}{11}$ c $\frac{\sqrt{2}}{2}$
d $\frac{\sqrt{5}}{5}$ e $\frac{1}{2}$ f $\frac{1}{4}$
g $\frac{\sqrt{13}}{13}$ h $\frac{1}{3}$

2 a $\frac{1 - \sqrt{3}}{-2}$ b $\sqrt{5} - 2$ c $\frac{3 + \sqrt{7}}{2}$
d $3 + \sqrt{5}$ e $\frac{\sqrt{5} + \sqrt{3}}{2}$ f $\frac{(3 - \sqrt{2})(4 + \sqrt{5})}{11}$
g $5(\sqrt{5} - 2)$ h $5(4 + \sqrt{14})$ i $\frac{11(3 - \sqrt{11})}{-2}$
j $\frac{5 - \sqrt{21}}{-2}$ k $\frac{14 - \sqrt{187}}{3}$ l $\frac{35 + \sqrt{1189}}{6}$
m -1

3 a $\frac{11 + 6\sqrt{2}}{49}$ b $9 - 4\sqrt{5}$ c $\frac{44 + 24\sqrt{2}}{49}$
d $\frac{81 - 30\sqrt{2}}{529}$ e $\frac{13 + 2\sqrt{2}}{161}$ f $\frac{7 - 3\sqrt{3}}{11}$

4 $-\frac{7}{4} + \frac{\sqrt{5}}{4}$

Answers

Mixed exercise

1 **a** y^8 **b** $6x^7$ **c** $32x$ **d** $12b^9$

2 **a** $x^2 - 2x - 15$ **b** $6x^2 - 19x - 7$
c $6x^2 - 2xy + 19x - 5y + 10$

3 **a** $x^3 + 3x^2 - 4x$ **b** $x^3 + 6x^2 - 13x - 42$
c $6x^3 - 5x^2 - 17x + 6$

4 **a** $15y + 12$ **b** $15x^2 - 25x^3 + 10x^4$
c $16x^2 + 13x$ **d** $9x^3 - 3x^2 + 4x$

5 **a** $x(3x + 4)$ **b** $2y(2y + 5)$
c $x(x + y + y^2)$ **d** $2xy(4y + 5x)$

6 **a** $(x + 1)(x + 2)$ **b** $3x(x + 2)$
c $(x - 7)(x + 5)$ **d** $(2x - 3)(x + 1)$
e $(5x + 2)(x - 3)$ **f** $(1 - x)(6 + x)$

7 **a** $2x(x^2 + 3)$ **b** $x(x + 6)(x - 6)$
c $x(2x - 3)(x + 5)$

8 **a** $3x^6$ **b** 2 **c** $6x^2$ **d** $\frac{1}{2}x^{-\frac{1}{3}}$

9 **a** $\frac{4}{9}$ **b** $\pm\frac{3375}{4913}$

10 **a** $\frac{\sqrt{7}}{7}$ **b** $4\sqrt{5}$

11 **a** 21 877
b $(5x + 6)(7x - 8)$
When $x = 25$, $5x + 6 = 131$ and $7x - 8 = 167$; both 131 and 167 are prime numbers.

12 **a** $3\sqrt{2} + \sqrt{10}$ **b** $10 + 2\sqrt{3} - 5\sqrt{5} - \sqrt{15}$
c $24 - 6\sqrt{7} - 4\sqrt{2} + \sqrt{14}$

13 **a** $\frac{\sqrt{3}}{3}$ **b** $\sqrt{2} + 1$ **c** $-3\sqrt{3} - 6$
d $\frac{30 - \sqrt{851}}{-7}$ **e** $7 - 4\sqrt{3}$ **f** $\frac{23 + 8\sqrt{7}}{81}$

14 **a** $b = -4$ and $c = -5$ **b** $(x + 3)(x - 5)(x + 1)$

15 **a** $\frac{1}{4}x$ **b** $256x^{-3}$

16 $\frac{5}{\sqrt{75} - \sqrt{50}} = \frac{1}{\sqrt{3} - \sqrt{2}} = \sqrt{3} + \sqrt{2}$

17 $-36 + 10\sqrt{11}$

18 $x(1 + 8x)(1 - 8x)$

19 $y = 6x + 3$

20 $4\sqrt{3}$

21 $3 - \sqrt{3}$ cm

22 $\frac{4 - 4x^{\frac{1}{2}} + x^1}{x^{\frac{1}{2}}} = 4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$

23 $\frac{11}{2}$

24 $4x^{\frac{5}{2}} + x^2$, $a = \frac{5}{2}$ $b = 2$

Challenge

a $a - b$

b $\frac{(\sqrt{1} - \sqrt{2}) + (\sqrt{2} - \sqrt{3}) + \ldots + (\sqrt{24} - \sqrt{25})}{-1} = \sqrt{25} - \sqrt{1} = 4$