

Southfields Academy A Level Mathematics Year 12 – Launchpad HW

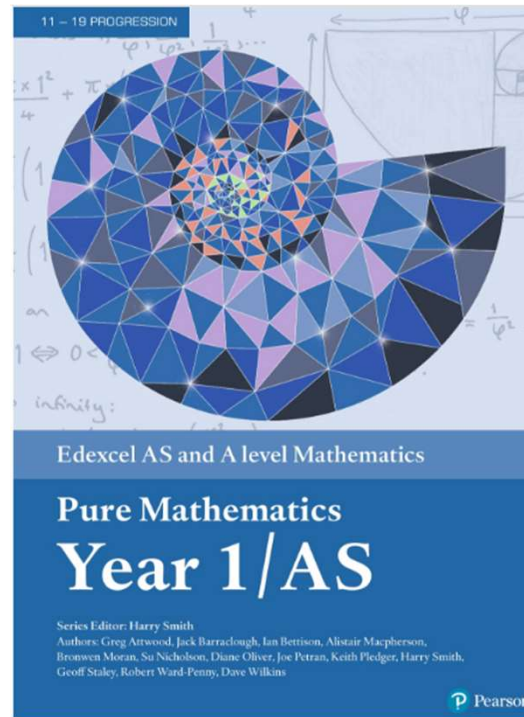
Task:

- Read the examples and questions in chapter 1.
- **You only have to answer every question from the MIXED EXERCISE (Q1 - Q24 + challenge Qs).**
- The answers are provided.
- Mark your own work.
- Bring your marked answers to your first maths lesson with any issues/questions for your teacher.

If you have any questions please email Ms.

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Prior knowledge check

- Simplify:
 - $4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$
 - $3x^2 - 5x + 2 + 3x^2 - 7x - 12$ ← GCSE Mathematics
- Write as a single power of 2:
 - $2^5 \times 2^3$
 - $2^6 \div 2^2$
 - $(2^3)^2$ ← GCSE Mathematics
- Expand:
 - $3(x + 4)$
 - $5(2 - 3x)$
 - $6(2x - 5y)$ ← GCSE Mathematics
- Write down the highest common factor of:
 - 24 and 16
 - $6x$ and $8x^2$
 - $4xy^2$ and $3xy$ ← GCSE Mathematics
- Simplify:
 - $\frac{10x}{5}$
 - $\frac{20x}{2}$
 - $\frac{40x}{24}$ ← GCSE Mathematics

Computer scientists use indices to describe very large numbers. A quantum computer with 1000 qubits (quantum bits) can consider 2^{1000} values simultaneously. This is greater than the number of particles in the observable universe.

1.1 Index laws

You can use the laws of indices to simplify powers of the same base.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$

Notation

x^5 This is the **base**.

This is the **index, power or exponent**.

Example 1

Simplify these expressions:

a $x^2 \times x^5$ b $2r^2 \times 3r^3$ c $\frac{b^7}{b^4}$ d $6x^5 \div 3x^3$ e $(a^3)^2 \times 2a^2$ f $(3x^2)^3 \div x^4$

a $x^2 \times x^5 = x^{2+5} = x^7$

Use the rule $a^m \times a^n = a^{m+n}$ to simplify the index.

b $2r^2 \times 3r^3 = 2 \times 3 \times r^2 \times r^3$
 $= 6 \times r^{2+3} = 6r^5$

Rewrite the expression with the numbers together and the r terms together.

c $\frac{b^7}{b^4} = b^{7-4} = b^3$

$2 \times 3 = 6$
 $r^2 \times r^3 = r^{2+3}$

d $6x^5 \div 3x^3 = \frac{6}{3} \times \frac{x^5}{x^3}$
 $= 2 \times x^2 = 2x^2$

Use the rule $a^m \div a^n = a^{m-n}$ to simplify the index.

$x^5 \div x^3 = x^{5-3} = x^2$

e $(a^3)^2 \times 2a^2 = a^6 \times 2a^2$
 $= 2 \times a^6 \times a^2 = 2a^8$

Use the rule $(a^m)^n = a^{mn}$ to simplify the index.

$a^6 \times a^2 = a^{6+2} = a^8$

f $\frac{(3x^2)^3}{x^4} = 3^3 \times \frac{(x^2)^3}{x^4}$
 $= 27 \times \frac{x^6}{x^4} = 27x^2$

Use the rule $(ab)^n = a^n b^n$ to simplify the numerator.

$(x^2)^3 = x^{2 \times 3} = x^6$

$\frac{x^6}{x^4} = x^{6-4} = x^2$

Exercise 1A

1 Simplify these expressions:

a $x^3 \times x^4$

b $2x^3 \times 3x^2$

c $\frac{k^3}{k^2}$

d $\frac{4p^3}{2p}$

e $\frac{3x^3}{3x^2}$

f $(y^2)^5$

g $10x^5 \div 2x^3$

h $(p^3)^2 \div p^4$

i $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$

k $2a^4 \times 3a^5$

l $\frac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$

n $3x^3 \times 2x^2 \times 4x^6$

o $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$

q $2a^3 \div 3a^2 \times 6a^5$

r $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x - 2)$

b $x(x + 9)$

c $-3y(4 - 3y)$

d $x(y + 5)$

e $-x(3x + 5)$

f $-5x(4x + 1)$

g $(4x + 5)x$

h $-3y(5 - 2y^2)$

i $-2x(5x - 4)$

j $(3x - 5)x^2$

k $3(x + 2) + (x - 7)$

l $5x - 6 - (3x - 2)$

m $4(c + 3d^2) - 3(2c + d^2)$

n $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

o $x(3x^2 - 2x + 5)$

p $7y^2(2 - 5y + 3y^2)$

q $-2y^2(5 - 7y + 3y^2)$

r $7(x - 2) + 3(x + 4) - 6(x - 2)$

s $5x - 3(4 - 2x) + 6$

t $3x^2 - x(3 - 4x) + 7$

u $4x(x + 3) - 2x(3x - 7)$

v $3x^2(2x + 1) - 5x^2(3x - 4)$

3 Simplify these fractions:

a $\frac{6x^4 + 10x^6}{2x}$

d $\frac{8x^3 + 5x}{2x}$

b $\frac{3x^5 - x^7}{x}$

e $\frac{7x^7 + 5x^2}{5x}$

c $\frac{2x^4 - 4x^2}{4x}$

f $\frac{9x^5 - 5x^3}{3x}$

Example 5

Expand these expressions and simplify if possible:

a $x(2x + 3)(x - 7)$

b $x(5x - 3y)(2x - y + 4)$

c $(x - 4)(x + 3)(x + 1)$

a $x(2x + 3)(x - 7)$
 $= (2x^2 + 3x)(x - 7)$
 $= 2x^3 - 14x^2 + 3x^2 - 21x$
 $= 2x^3 - 11x^2 - 21x$

b $x(5x - 3y)(2x - y + 4)$
 $= (5x^2 - 3xy)(2x - y + 4)$
 $= 5x^2(2x - y + 4) - 3xy(2x - y + 4)$
 $= 10x^3 - 5x^2y + 20x^2 - 6x^2y + 3xy^2 - 12xy$
 $= 10x^3 - 11x^2y + 20x^2 + 3xy^2 - 12xy$

c $(x - 4)(x + 3)(x + 1)$
 $= (x^2 - x - 12)(x + 1)$
 $= x^2(x + 1) - x(x + 1) - 12(x + 1)$
 $= x^3 + x^2 - x^2 - x - 12x - 12$
 $= x^3 - 13x - 12$

Start by expanding one pair of brackets:
 $x(2x + 3) = 2x^2 + 3x$

You could also have expanded the second pair of brackets first: $(2x + 3)(x - 7) = 2x^2 - 11x - 21$
Then multiply by x .

Be careful with minus signs. You need to change every sign in the second pair of brackets when you multiply it out.

Choose one pair of brackets to expand first, for example:

$$(x - 4)(x + 3) = x^2 + 3x - 4x - 12$$
$$= x^2 - x - 12$$

You multiplied together three linear terms, so the final answer contains an x^3 term.

Exercise 1B

1 Expand and simplify if possible:

a $(x + 4)(x + 7)$

b $(x - 3)(x + 2)$

c $(x - 2)^2$

d $(x - y)(2x + 3)$

e $(x + 3y)(4x - y)$

f $(2x - 4y)(3x + y)$

g $(2x - 3)(x - 4)$

h $(3x + 2y)^2$

i $(2x + 8y)(2x + 3)$

j $(x + 5)(2x + 3y - 5)$

k $(x - 1)(3x - 4y - 5)$

l $(x - 4y)(2x + y + 5)$

m $(x + 2y - 1)(x + 3)$

n $(2x + 2y + 3)(x + 6)$

o $(4 - y)(4y - x + 3)$

p $(4y + 5)(3x - y + 2)$

q $(5y - 2x + 3)(x - 4)$

r $(4y - x - 2)(5 - y)$

2 Expand and simplify if possible:

a $5(x + 1)(x - 4)$

b $7(x - 2)(2x + 5)$

c $3(x - 3)(x - 3)$

d $x(x - y)(x + y)$

e $x(2x + y)(3x + 4)$

f $y(x - 5)(x + 1)$

g $y(3x - 2y)(4x + 2)$

h $y(7 - x)(2x - 5)$

i $x(2x + y)(5x - 2)$

j $x(x + 2)(x + 3y - 4)$

k $y(2x + y - 1)(x + 5)$

l $y(3x + 2y - 3)(2x + 1)$

m $x(2x + 3)(x + y - 5)$

n $2x(3x - 1)(4x - y - 3)$

o $3x(x - 2y)(2x + 3y + 5)$

p $(x + 3)(x + 2)(x + 1)$

q $(x + 2)(x - 4)(x + 3)$

r $(x + 3)(x - 1)(x - 5)$

s $(x - 5)(x - 4)(x - 3)$

t $(2x + 1)(x - 2)(x + 1)$

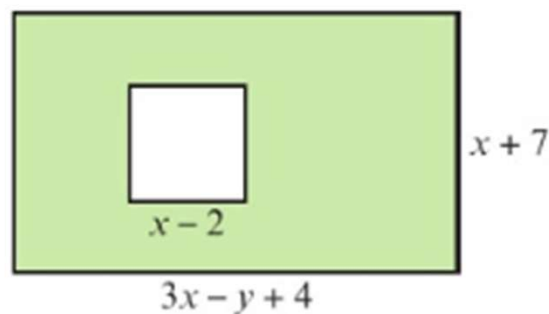
u $(2x + 3)(3x - 1)(x + 2)$

v $(3x - 2)(2x + 1)(3x - 2)$

w $(x + y)(x - y)(x - 1)$

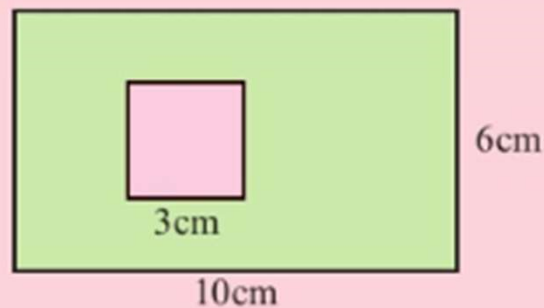
x $(2x - 3y)^3$

- 3 The diagram shows a rectangle with a square cut out. The rectangle has length $3x - y + 4$ and width $x + 7$. The square has length $x - 2$. Find an expanded and simplified expression for the shaded area.



Problem-solving

Use the same strategy as you would use if the lengths were given as numbers:



- 4 A cuboid has dimensions $x + 2$ cm, $2x - 1$ cm and $2x + 3$ cm. Show that the volume of the cuboid is $4x^3 + 12x^2 + 5x - 6$ cm³.
- 5 Given that $(2x + 5y)(3x - y)(2x + y) = ax^3 + bx^2y + cxy^2 + dy^3$, where a , b , c and d are constants, find the values of a , b , c and d . (2 marks)

Challenge

Expand and simplify $(x + y)^4$.

Links

You can use the binomial expansion to expand expressions like $(x + y)^4$ quickly. → Section 8.3

Example**7**

Factorise:

a $x^2 - 5x - 6$

b $x^2 + 6x + 8$

c $6x^2 - 11x - 10$

d $x^2 - 25$

e $4x^2 - 9y^2$

- A quadratic expression has the form $ax^2 + bx + c$ where a , b and c are real numbers and $a \neq 0$.

a $x^2 - 5x - 6$

$ac = -6$ and $b = -5$

So $x^2 - 5x - 6 = x^2 + x - 6x - 6$

$= x(x + 1) - 6(x + 1)$

$= (x + 1)(x - 6)$

c $6x^2 - 11x - 10$

$= 6x^2 - 15x + 4x - 10$

$= 3x(2x - 5) + 2(2x - 5)$

$= (2x - 5)(3x + 2)$

d $x^2 - 25$

$= x^2 - 5^2$

$= (x + 5)(x - 5)$

e $4x^2 - 9y^2$

$= 2^2x^2 - 3^2y^2$

$= (2x + 3y)(2x - 3y)$

Exercise 1C

1 Factorise these expressions completely:

1 Factorise these expressions completely:

a $4x + 8$

b $6x - 24$

c $20x + 15$

d $2x^2 + 4$

e $4x^2 + 20$

f $6x^2 - 18x$

g $x^2 - 7x$

h $2x^2 + 4x$

i $3x^2 - x$

j $6x^2 - 2x$

k $10y^2 - 5y$

l $35x^2 - 28x$

m $x^2 + 2x$

n $3y^2 + 2y$

o $4x^2 + 12x$

p $5y^2 - 20y$

q $9xy^2 + 12x^2y$

r $6ab - 2ab^2$

s $5x^2 - 25xy$

t $12x^2y + 8xy^2$

u $15y - 20yz^2$

v $12x^2 - 30$

w $xy^2 - x^2y$

x $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$

b $2x^2 + 6x$

c $x^2 + 11x + 24$

d $x^2 + 8x + 12$

e $x^2 + 3x - 40$

f $x^2 - 8x + 12$

g $x^2 + 5x + 6$

h $x^2 - 2x - 24$

i $x^2 - 3x - 10$

j $x^2 + x - 20$

k $2x^2 + 5x + 2$

l $3x^2 + 10x - 8$

m $5x^2 - 16x + 3$

n $6x^2 - 8x - 8$

o $2x^2 + 7x - 15$

p $2x^4 + 14x^2 + 24$

q $x^2 - 4$

r $x^2 - 49$

s $4x^2 - 25$

t $9x^2 - 25y^2$

v $2x^2 - 50$

w $6x^2 - 10x + 4$

u $36x^2 - 4$

x $15x^2 + 42x - 9$

Hint For part **n**, take 2 out first. For part **p**, let $u = x^2$.

3 Factorise completely:

a $x^3 + 2x$

d $x^3 - 9x$

g $x^3 - 7x^2 + 6x$

j $2x^3 + 13x^2 + 15x$

b $x^3 - x^2 + x$

e $x^3 - x^2 - 12x$

h $x^3 - 64x$

k $x^3 - 4x$

c $x^3 - 5x$

f $x^3 + 11x^2 + 30x$

i $2x^3 - 5x^2 - 3x$

l $3x^3 + 27x^2 + 60x$

4 Factorise completely $x^4 - y^4$. (2 marks)

Problem-solving

Watch out for terms that can be written as a function of a function: $x^4 = (x^2)^2$

5 Factorise completely $6x^3 + 7x^2 - 5x$. (2 marks)

Challenge

Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

1.4 Negative and fractional indices

■ You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$

- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$

- $a^{-m} = \frac{1}{a^m}$

- $a^0 = 1$

Example 9

Simplify:

a $\frac{x^3}{x^{-3}}$
 b $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$
 c $(x^3)^{\frac{2}{3}}$
 d $2x^{1.5} \div 4x^{-0.25}$
 e $\sqrt[3]{125x^6}$
 f $\frac{2x^2 - x}{x^5}$

$$\text{a } \frac{x^3}{x^{-3}} = x^{3 - (-3)} = x^6$$

Use the rule $a^m \div a^n = a^{m-n}$.

$$\text{b } x^{\frac{1}{2}} \times x^{\frac{3}{2}} = x^{\frac{1}{2} + \frac{3}{2}} = x^2$$

This could also be written as \sqrt{x} .

Use the rule $a^m \times a^n = a^{m+n}$.

$$\text{c } (x^3)^{\frac{2}{3}} = x^{3 \times \frac{2}{3}} = x^2$$

Use the rule $(a^m)^n = a^{mn}$.

$$\text{d } 2x^{1.5} \div 4x^{-0.25} = \frac{1}{2}x^{1.5 - (-0.25)} = \frac{1}{2}x^{1.75}$$

Use the rule $a^m \div a^n = a^{m-n}$.

$1.5 - (-0.25) = 1.75$

$$\begin{aligned} \text{e } \sqrt[3]{125x^6} &= (125x^6)^{\frac{1}{3}} \\ &= (125)^{\frac{1}{3}}(x^6)^{\frac{1}{3}} = \sqrt[3]{125}(x^{6 \times \frac{1}{3}}) = 5x^2 \end{aligned}$$

Using $a^{\frac{1}{m}} = \sqrt[m]{a}$.

$$\begin{aligned} \text{f } \frac{2x^2 - x}{x^5} &= \frac{2x^2}{x^5} - \frac{x}{x^5} \\ &= 2 \times x^{2-5} - x^{1-5} = 2x^{-3} - x^{-4} \\ &= \frac{2}{x^3} - \frac{1}{x^4} \end{aligned}$$

Divide each term of the numerator by x^5 .

Using $a^{-m} = \frac{1}{a^m}$

Example 11

Given that $y = \frac{1}{16}x^2$ express each of the following in the form kx^n , where k and n are constants.

a $y^{\frac{1}{2}}$

b $4y^{-1}$

$$\begin{aligned} \text{a } y^{\frac{1}{2}} &= \left(\frac{1}{16}x^2\right)^{\frac{1}{2}} \\ &= \frac{1}{\sqrt{16}}x^{2 \times \frac{1}{2}} = \frac{x}{4} \end{aligned}$$

$$\begin{aligned} \text{b } 4y^{-1} &= 4\left(\frac{1}{16}x^2\right)^{-1} \\ &= 4\left(\frac{1}{16}\right)^{-1}x^{2 \times (-1)} = 4 \times 16x^{-2} \\ &= 64x^{-2} \end{aligned}$$

Substitute $y = \frac{1}{16}x^2$ into $y^{\frac{1}{2}}$.

$$\left(\frac{1}{16}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{16}} \text{ and } (x^2)^{\frac{1}{2}} = x^{2 \times \frac{1}{2}}$$

$$\left(\frac{1}{16}\right)^{-1} = 16 \text{ and } x^{2 \times (-1)} = x^{-2}$$

Problem-solving

Check that your answers are in the correct form. If k and n are constants they could be positive or negative, and they could be integers, fractions or surds.

Exercise 1D

1 Simplify:

a $x^3 \div x^{-2}$

d $(x^2)^{\frac{3}{2}}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$

j $\sqrt{x} \times \sqrt[3]{x}$

b $x^5 \div x^7$

e $(x^3)^{\frac{5}{3}}$

h $5x^{\frac{7}{3}} \div x^{\frac{2}{3}}$

k $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$

c $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

f $3x^{0.5} \times 4x^{-0.5}$

i $3x^4 \times 2x^{-5}$

l $\frac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$

d 4^{-2}

g $(\frac{3}{4})^0$

j $(\frac{27}{8})^{\frac{2}{3}}$

b $81^{\frac{3}{2}}$

e $9^{-\frac{1}{2}}$

h $1296^{\frac{3}{4}}$

k $(\frac{6}{5})^{-1}$

c $27^{\frac{1}{3}}$

f $(-5)^{-3}$

i $(\frac{25}{16})^{\frac{3}{2}}$

l $(\frac{343}{512})^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$

b $\frac{5x^3 - 2x^2}{x^5}$

c $(125x^{12})^{\frac{1}{3}}$

d $\frac{x + 4x^3}{x^3}$

e $\frac{2x + x^2}{x^4}$

f $\left(\frac{4}{9}x^4\right)^{\frac{3}{2}}$

g $\frac{9x^2 - 15x^5}{3x^3}$

h $\frac{5x + 3x^2}{15x^3}$

4 a Find the value of $81^{\frac{1}{4}}$.

(1 mark)

b Simplify $x(2x^{-\frac{1}{3}})^4$.

(2 marks)

5 Given that $y = \frac{1}{8}x^3$ express each of the following in the form kx^n , where k and n are constants.

a $y^{\frac{1}{3}}$

(2 marks)

b $\frac{1}{2}y^{-2}$

(2 marks)

1.5 Surds

If n is an integer that is **not** a square number, then any multiple of \sqrt{n} is called a surd.

Examples of surds are $\sqrt{2}$, $\sqrt{19}$ and $5\sqrt{2}$.

Surds are examples of **irrational numbers**.

The decimal expansion of a surd is never-ending and never repeats, for example $\sqrt{2} = 1.414213562\dots$

You can use surds to write exact answers to calculations.

■ You can manipulate surds using these rules:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Notation Irrational numbers cannot be written in the form $\frac{a}{b}$ where a and b are integers. Surds are examples of **irrational numbers**.

Example 13

Expand and simplify if possible:

a $\sqrt{2}(5 - \sqrt{3})$

b $(2 - \sqrt{3})(5 + \sqrt{3})$

$$\begin{aligned} \text{a } & \sqrt{2}(5 - \sqrt{3}) \\ &= 5\sqrt{2} - \sqrt{2}\sqrt{3} \\ &= 5\sqrt{2} - \sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{b } & (2 - \sqrt{3})(5 + \sqrt{3}) \\ &= 2(5 + \sqrt{3}) - \sqrt{3}(5 + \sqrt{3}) \\ &= 10 + 2\sqrt{3} - 5\sqrt{3} - \sqrt{9} \\ &= 7 - 3\sqrt{3} \end{aligned}$$

Exercise 1E

1 Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$

b $\sqrt{72}$

c $\sqrt{50}$

d $\sqrt{32}$

e $\sqrt{90}$

f $\frac{\sqrt{12}}{2}$

g $\frac{\sqrt{27}}{3}$

h $\sqrt{20} + \sqrt{80}$

i $\sqrt{200} + \sqrt{18} - \sqrt{72}$

j $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$

k $\sqrt{28} - 2\sqrt{63} + \sqrt{7}$

l $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

m $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$

n $\frac{\sqrt{44}}{\sqrt{11}}$

o $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2 + \sqrt{3})$

b $\sqrt{5}(3 - \sqrt{3})$

c $\sqrt{2}(4 - \sqrt{5})$

d $(2 - \sqrt{2})(3 + \sqrt{5})$

e $(2 - \sqrt{3})(3 - \sqrt{7})$

f $(4 + \sqrt{5})(2 + \sqrt{5})$

g $(5 - \sqrt{3})(1 - \sqrt{3})$

h $(4 + \sqrt{3})(2 - \sqrt{3})$

i $(7 - \sqrt{11})(2 + \sqrt{11})$

3 Simplify $\sqrt{75} - \sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer.

1.6 Rationalising denominators

If a fraction has a surd in the denominator, it is sometimes useful to **rearrange** it so that the denominator is a **rational** number. This is called rationalising the denominator.

■ **The rules to rationalise denominators are:**

- For fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by \sqrt{a} .
- For fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $a - \sqrt{b}$.
- For fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.

Example 14

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$

b $\frac{1}{3 + \sqrt{2}}$

c $\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$

d $\frac{1}{(1 - \sqrt{3})^2}$

$$\begin{aligned} \text{a } \frac{1}{\sqrt{3}} &= \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \frac{\sqrt{3}}{3} \end{aligned}$$

Multiply the numerator and denominator by $\sqrt{3}$.

$$\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$$

$$\begin{aligned} \text{b } \frac{1}{3 + \sqrt{2}} &= \frac{1 \times (3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} \\ &= \frac{3 - \sqrt{2}}{9 - 3\sqrt{2} + 3\sqrt{2} - 2} \\ &= \frac{3 - \sqrt{2}}{7} \end{aligned}$$

Multiply numerator and denominator by $(3 - \sqrt{2})$.

$$\sqrt{2} \times \sqrt{2} = 2$$

$$9 - 2 = 7, -3\sqrt{2} + 3\sqrt{2} = 0$$

$$c \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{(\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})}$$

$$= \frac{5 + \sqrt{5}\sqrt{2} + \sqrt{2}\sqrt{5} + 2}{5 - 2}$$

$$= \frac{7 + 2\sqrt{10}}{3}$$

Multiply numerator and denominator by $\sqrt{5} + \sqrt{2}$.

$-\sqrt{2}\sqrt{5}$ and $\sqrt{5}\sqrt{2}$ cancel each other out.

$$\sqrt{5}\sqrt{2} = \sqrt{10}$$

$$d \frac{1}{(1 - \sqrt{3})^2} = \frac{1}{(1 - \sqrt{3})(1 - \sqrt{3})}$$

$$= \frac{1}{1 - \sqrt{3} - \sqrt{3} + \sqrt{9}}$$

$$= \frac{1}{4 - 2\sqrt{3}}$$

$$= \frac{1 \times (4 + 2\sqrt{3})}{(4 - 2\sqrt{3})(4 + 2\sqrt{3})}$$

$$= \frac{4 + 2\sqrt{3}}{16 + 8\sqrt{3} - 8\sqrt{3} - 12}$$

$$= \frac{4 + 2\sqrt{3}}{4} = \frac{2 + \sqrt{3}}{2}$$

Expand the brackets.

Simplify and collect like terms. $\sqrt{9} = 3$

Multiply the numerator and denominator by $4 + 2\sqrt{3}$.

$$\sqrt{3} \times \sqrt{3} = 3$$

$$16 - 12 = 4, 8\sqrt{3} - 8\sqrt{3} = 0$$

Exercise 1F**1 Simplify:**

a $\frac{1}{\sqrt{5}}$

b $\frac{1}{\sqrt{11}}$

c $\frac{1}{\sqrt{2}}$

d $\frac{\sqrt{3}}{\sqrt{15}}$

e $\frac{\sqrt{12}}{\sqrt{48}}$

f $\frac{\sqrt{5}}{\sqrt{80}}$

g $\frac{\sqrt{12}}{\sqrt{156}}$

h $\frac{\sqrt{7}}{\sqrt{63}}$

2 Rationalise the denominators and simplify:

a $\frac{1}{1 + \sqrt{3}}$

b $\frac{1}{2 + \sqrt{5}}$

c $\frac{1}{3 - \sqrt{7}}$

d $\frac{4}{3 - \sqrt{5}}$

e $\frac{1}{\sqrt{5} - \sqrt{3}}$

f $\frac{3 - \sqrt{2}}{4 - \sqrt{5}}$

g $\frac{5}{2 + \sqrt{5}}$

h $\frac{5\sqrt{2}}{\sqrt{8} - \sqrt{7}}$

i $\frac{11}{3 + \sqrt{11}}$

j $\frac{\sqrt{3} - \sqrt{7}}{\sqrt{3} + \sqrt{7}}$

k $\frac{\sqrt{17} - \sqrt{11}}{\sqrt{17} + \sqrt{11}}$

l $\frac{\sqrt{41} + \sqrt{29}}{\sqrt{41} - \sqrt{29}}$

m $\frac{\sqrt{2} - \sqrt{3}}{\sqrt{3} - \sqrt{2}}$

3 Rationalise the denominators and simplify:

a $\frac{1}{(3 - \sqrt{2})^2}$

b $\frac{1}{(2 + \sqrt{5})^2}$

c $\frac{4}{(3 - \sqrt{2})^2}$

d $\frac{3}{(5 + \sqrt{2})^2}$

e $\frac{1}{(5 + \sqrt{2})(3 - \sqrt{2})}$

f $\frac{2}{(5 - \sqrt{3})(2 + \sqrt{3})}$

4 Simplify $\frac{3 - 2\sqrt{5}}{\sqrt{5} - 1}$ giving your answer in the form $p + q\sqrt{5}$, where p and q are rational numbers. **(4 marks)**

Problem-solving

You can check that your answer is in the correct form by writing down the values of p and q and checking that they are rational numbers.

Mixed exercise 1

1 Simplify:

a $y^3 \times y^5$

b $3x^2 \times 2x^5$

c $(4x^2)^3 \div 2x^5$

d $4b^2 \times 3b^3 \times b^4$

2 Expand and simplify if possible:

a $(x + 3)(x - 5)$

b $(2x - 7)(3x + 1)$

c $(2x + 5)(3x - y + 2)$

3 Expand and simplify if possible:

a $x(x + 4)(x - 1)$

b $(x + 2)(x - 3)(x + 7)$

c $(2x + 3)(x - 2)(3x - 1)$

4 Expand the brackets:

a $3(5y + 4)$

b $5x^2(3 - 5x + 2x^2)$

c $5x(2x + 3) - 2x(1 - 3x)$

d $3x^2(1 + 3x) - 2x(3x - 2)$

5 Factorise these expressions completely:

a $3x^2 + 4x$

b $4y^2 + 10y$

c $x^2 + xy + xy^2$

d $8xy^2 + 10x^2y$

6 Factorise:

a $x^2 + 3x + 2$

b $3x^2 + 6x$

c $x^2 - 2x - 35$

d $2x^2 - x - 3$

e $5x^2 - 13x - 6$

f $6 - 5x - x^2$

7 Factorise:

a $2x^3 + 6x$

b $x^3 - 36x$

c $2x^3 + 7x^2 - 15x$

8 Simplify:

a $9x^3 \div 3x^{-3}$

b $(4^{\frac{3}{2}})^{\frac{1}{3}}$

c $3x^{-2} \times 2x^4$

d $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

9 Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$

b $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify:

a $\frac{3}{\sqrt{63}}$

b $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

- 11 a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.
- b** By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.
- 12** Expand and simplify if possible:
- a** $\sqrt{2}(3 + \sqrt{5})$ **b** $(2 - \sqrt{5})(5 + \sqrt{3})$ **c** $(6 - \sqrt{2})(4 - \sqrt{7})$
- 13** Rationalise the denominator and simplify:
- a** $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2} - 1}$ **c** $\frac{3}{\sqrt{3} - 2}$ **d** $\frac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$ **e** $\frac{1}{(2 + \sqrt{3})^2}$ **f** $\frac{1}{(4 - \sqrt{7})^2}$
- 14 a** Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c .
- b** Hence, fully factorise $x^3 - x^2 - 17x - 15$.
- 15** Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n , where k and n are constants.
- a** $y^{\frac{1}{3}}$ **(1 mark)**
- b** $4y^{-1}$ **(1 mark)**

- 16** Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(5 marks)**
- 17** Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$. **(2 marks)**
- 18** Factorise completely $x - 64x^3$. **(3 marks)**
- 19** Express 27^{2x+1} in the form 3^y , stating y in terms of x . **(2 marks)**

20 Solve the equation $8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

(4 marks)

21 A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm².
Calculate the width of the rectangle in cm.

Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

22 Show that $\frac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$.

(2 marks)

23 Given that $243\sqrt{3} = 3^a$, find the value of a .

(3 marks)

24 Given that $\frac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a
and the value of b .

(2 marks)

Challenge

a Simplify $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$.

b Hence show that $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{24} + \sqrt{25}} = 4$

Summary of key points

1 You can use the laws of indices to simplify powers of the **same base**.

- $a^m \times a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $a^m \div a^n = a^{m-n}$
- $(ab)^n = a^n b^n$

2 Factorising is the opposite of expanding brackets.

3 A quadratic expression has the form $ax^2 + bx + c$ where a , b and c are real numbers and $a \neq 0$.

4 $x^2 - y^2 = (x + y)(x - y)$

5 You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{-m} = \frac{1}{a^m}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^0 = 1$

6 You can manipulate surds using these rules:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

7 The rules to rationalise denominators are:

- Fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by \sqrt{a} .
- Fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $a - \sqrt{b}$.
- Fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.

Answers

Exercise 1B

- 1 a $x^2 + 11x + 28$
b $x^2 - x - 6$
c $x^2 - 4x + 4$
d $2x^2 + 3x - 2xy - 3y$
e $4x^2 + 11xy - 3y^2$
f $6x^2 - 10xy - 4y^2$
g $2x^2 - 11x + 12$
h $9x^2 + 12xy + 4y^2$
i $4x^2 + 6x + 16xy + 24y$
j $2x^2 + 3xy + 5x + 15y - 2$
k $3x^2 - 4xy - 8x + 4y + 5$
l $2x^2 + 5x - 7xy - 4y^2 - 20y$
m $x^2 + 2x + 2xy + 6y - 3$
n $2x^2 + 15x + 2xy + 12y + 18$
o $13y - 4x + 12 - 4y^2 + xy$
p $12xy - 4y^2 + 3y + 15x + 10$
q $5xy - 20y - 2x^2 + 11x - 12$
r $22y - 4y^2 - 5x + xy - 10$
- 2 a $5x^2 - 15x - 20$
b $14x^2 + 7x - 70$
c $3x^2 - 18x + 27$
d $x^3 - xy^2$
e $6x^3 + 8x^2 + 3x^2y + 4xy$
f $x^2y - 4xy - 5y$
g $12x^2y + 6xy - 8xy^2 - 4y^2$
h $19xy - 35y - 2x^2y$
i $10x^3 - 4x^2 + 5x^2y - 2xy$
j $x^3 + 3x^2y - 2x^2 + 6xy - 8x$
- k $2x^2y + 9xy + xy^2 + 5y^2 - 5y$
l $6x^2y + 4xy^2 + 2y^2 - 3xy - 3y$
m $2x^3 + 2x^2y - 7x^2 + 3xy - 15x$
n $24x^3 - 6x^2y - 26x^2 + 2xy + 6x$
o $6x^3 + 15x^2 - 3x^2y - 18xy^2 - 30xy$
p $x^3 + 6x^2 + 11x + 6$
q $x^3 + x^2 - 14x - 24$
r $x^3 - 3x^2 - 13x + 15$
s $x^3 - 12x^2 + 47x - 60$
t $2x^3 - x^2 - 5x - 2$
u $6x^3 + 19x^2 + 11x - 6$
v $18x^3 - 15x^2 - 4x + 4$
w $x^3 - xy^2 - x^2 + y^2$
x $8x^3 - 36x^2y + 54xy^2 - 27y^3$
- 3 $2x^2 - xy + 29x - 7y + 24$
4 $4x^3 + 12x^2 + 5x - 6 \text{ cm}^3$
5 $a = 12, b = 32, c = 3, d = -5$

Challenge

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Answers

Exercise 1C

- 1 a $4(x + 2)$
c $5(4x + 3)$
e $4(x^2 + 5)$
g $x(x - 7)$
i $x(3x - 1)$
k $5y(2y - 1)$
m $x(x + 2)$
o $4x(x + 3)$
q $3xy(3y + 4x)$
s $5x(x - 5y)$
u $5y(3 - 4z^2)$
w $xy(y - x)$
- 2 a $x(x + 4)$
c $(x + 8)(x + 3)$
e $(x + 8)(x - 5)$
g $(x + 2)(x + 3)$
i $(x - 5)(x + 2)$
k $(2x + 1)(x + 2)$
m $(5x - 1)(x - 3)$
o $(2x - 3)(x + 5)$
q $(x + 2)(x - 2)$
s $(2x + 5)(2x - 5)$
u $4(3x + 1)(3x - 1)$
w $2(3x - 2)(x - 1)$
- b $6(x - 4)$
d $2(x^2 + 2)$
f $6x(x - 3)$
h $2x(x + 2)$
j $2x(3x - 1)$
l $7x(5x - 4)$
n $y(3y + 2)$
p $5y(y - 4)$
r $2ab(3 - b)$
t $4xy(3x + 2y)$
v $6(2x^2 - 5)$
x $4y(3y - x)$
- b $2x(x + 3)$
d $(x + 6)(x + 2)$
f $(x - 6)(x - 2)$
h $(x - 6)(x + 4)$
j $(x + 5)(x - 4)$
l $(3x - 2)(x + 4)$
n $2(3x + 2)(x - 2)$
p $2(x^2 + 3)(x^2 + 4)$
r $(x + 7)(x - 7)$
t $(3x + 5y)(3x - 5y)$
v $2(x + 5)(x - 5)$
x $3(5x - 1)(x + 3)$

- 3 a $x(x^2 + 2)$
c $x(x^2 - 5)$
e $x(x - 4)(x + 3)$
g $x(x - 1)(x - 6)$
i $x(2x + 1)(x - 3)$
k $x(x + 2)(x - 2)$
- b $x(x^2 - x + 1)$
d $x(x + 3)(x - 3)$
f $x(x + 5)(x + 6)$
h $x(x + 8)(x - 8)$
j $x(2x + 3)(x + 5)$
l $3x(x + 4)(x + 5)$
- 4 $(x^2 + y^2)(x + y)(x - y)$
- 5 $x(3x + 5)(2x - 1)$

Challenge

$$(x - 1)(x + 1)(2x + 3)(2x - 3)$$

Exercise 1D

- 1 a x^5 b x^{-2} c x^4 d x^3
e x^5 f $12x^0 = 12$ g $3x^{\frac{1}{2}}$ h $5x$
i $6x^{-1}$ j $x^{\frac{1}{3}}$ k $x^{\frac{17}{6}}$ l $x^{\frac{1}{6}}$

Answers

2 a 5 b 729 c 3 d $\frac{1}{16}$
 e $\frac{1}{3}$ f $\frac{-1}{125}$ g 1 h 216
 i $\frac{125}{64}$ j $\frac{9}{4}$ k $\frac{5}{6}$ l $\frac{64}{49}$

3 a $8x^5$ b $\frac{5}{x^2} - \frac{2}{x^3}$ c $5x^4$
 d $\frac{1}{x^2} + 4$ e $\frac{2}{x^3} + \frac{1}{x^2}$ f $\frac{8}{27}x^6$
 g $\frac{3}{x} - 5x^2$ h $\frac{1}{3x^2} + \frac{1}{5x}$

4 a 3 b $\frac{16}{\sqrt[3]{x}}$

5 a $\frac{x}{2}$ b $\frac{32}{x^6}$

Exercise 1E

1 a $2\sqrt{7}$ b $6\sqrt{2}$ c $5\sqrt{2}$ d $4\sqrt{2}$
 e $3\sqrt{10}$ f $\sqrt{3}$ g $\sqrt{3}$ h $6\sqrt{5}$
 i $7\sqrt{2}$ j $12\sqrt{7}$ k $-3\sqrt{7}$ l $9\sqrt{5}$
 m $23\sqrt{5}$ n 2 o $19\sqrt{3}$

2 a $2\sqrt{3} + 3$ b $3\sqrt{5} - \sqrt{15}$
 c $4\sqrt{2} - \sqrt{10}$ d $6 + 2\sqrt{5} - 3\sqrt{2} - \sqrt{10}$
 e $6 - 2\sqrt{7} - 3\sqrt{3} + \sqrt{21}$ f $13 + 6\sqrt{5}$
 g $8 - 6\sqrt{3}$ h $5 - 2\sqrt{3}$
 i $3 + 5\sqrt{11}$

3 $3\sqrt{3}$

Exercise 1F

1 a $\frac{\sqrt{5}}{5}$ b $\frac{\sqrt{11}}{11}$ c $\frac{\sqrt{2}}{2}$

d $\frac{\sqrt{5}}{5}$ e $\frac{1}{2}$ f $\frac{1}{4}$

g $\frac{\sqrt{13}}{13}$ h $\frac{1}{3}$

2 a $\frac{1 - \sqrt{3}}{-2}$ b $\sqrt{5} - 2$ c $\frac{3 + \sqrt{7}}{2}$

d $3 + \sqrt{5}$ e $\frac{\sqrt{5} + \sqrt{3}}{2}$ f $\frac{(3 - \sqrt{2})(4 + \sqrt{5})}{11}$

g $5(\sqrt{5} - 2)$ h $5(4 + \sqrt{14})$ i $\frac{11(3 - \sqrt{11})}{-2}$

j $\frac{5 - \sqrt{21}}{-2}$ k $\frac{14 - \sqrt{187}}{3}$ l $\frac{35 + \sqrt{1189}}{6}$

m -1

3 a $\frac{11 + 6\sqrt{2}}{49}$ b $9 - 4\sqrt{5}$ c $\frac{44 + 24\sqrt{2}}{49}$

d $\frac{81 - 30\sqrt{2}}{529}$ e $\frac{13 + 2\sqrt{2}}{161}$ f $\frac{7 - 3\sqrt{3}}{11}$

4 $-\frac{7}{4} + \frac{\sqrt{5}}{4}$

Answers

Mixed exercise

- 1 a y^8 b $6x^7$ c $32x$ d $12b^9$
- 2 a $x^2 - 2x - 15$ b $6x^2 - 19x - 7$
c $6x^2 - 2xy + 19x - 5y + 10$
- 3 a $x^3 + 3x^2 - 4x$ b $x^3 + 6x^2 - 13x - 42$
c $6x^3 - 5x^2 - 17x + 6$
- 4 a $15y + 12$ b $15x^2 - 25x^3 + 10x^4$
c $16x^2 + 13x$ d $9x^3 - 3x^2 + 4x$
- 5 a $x(3x + 4)$ b $2y(2y + 5)$
c $x(x + y + y^2)$ d $2xy(4y + 5x)$
- 6 a $(x + 1)(x + 2)$ b $3x(x + 2)$
c $(x - 7)(x + 5)$ d $(2x - 3)(x + 1)$
e $(5x + 2)(x - 3)$ f $(1 - x)(6 + x)$
- 7 a $2x(x^2 + 3)$ b $x(x + 6)(x - 6)$
c $x(2x - 3)(x + 5)$
- 8 a $3x^6$ b 2 c $6x^2$ d $\frac{1}{2}x^{-\frac{1}{3}}$
- 9 a $\frac{4}{9}$ b $\pm \frac{3375}{4913}$
- 10 a $\frac{\sqrt{7}}{7}$ b $4\sqrt{5}$
- 11 a 21877
b $(5x + 6)(7x - 8)$
When $x = 25$, $5x + 6 = 131$ and $7x - 8 = 167$; both 131 and 167 are prime numbers.

- 12 a $3\sqrt{2} + \sqrt{10}$ b $10 + 2\sqrt{3} - 5\sqrt{5} - \sqrt{15}$
c $24 - 6\sqrt{7} - 4\sqrt{2} + \sqrt{14}$
- 13 a $\frac{\sqrt{3}}{3}$ b $\sqrt{2} + 1$ c $-3\sqrt{3} - 6$
d $\frac{30 - \sqrt{851}}{-7}$ e $7 - 4\sqrt{3}$ f $\frac{23 + 8\sqrt{7}}{81}$
- 14 a $b = -4$ and $c = -5$ b $(x + 3)(x - 5)(x + 1)$
- 15 a $\frac{1}{4}x$ b $256x^{-3}$
- 16 $\frac{5}{\sqrt{75} - \sqrt{50}} = \frac{1}{\sqrt{3} - \sqrt{2}} = \sqrt{3} + \sqrt{2}$
- 17 $-36 + 10\sqrt{11}$
- 18 $x(1 + 8x)(1 - 8x)$
- 19 $y = 6x + 3$
- 20 $4\sqrt{3}$
- 21 $3 - \sqrt{3}$ cm
- 22 $\frac{4 - 4x^{\frac{1}{2}} + x^1}{x^{\frac{1}{2}}} = 4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$
- 23 $\frac{11}{2}$
- 24 $4x^{\frac{5}{2}} + x^2$, $a = \frac{5}{2}$ $b = 2$

Challenge

- a $a - b$
- b $\frac{(\sqrt{1} - \sqrt{2}) + (\sqrt{2} - \sqrt{3}) + \dots + (\sqrt{24} - \sqrt{25})}{-1} = \sqrt{25} - \sqrt{1} = 4$