Y12 A-Level Mathematics Entrance Exam Revision Homework

Please complete all of these questions in preparation for your entrance exam.
DO NOT use a calculator, the exam will be a non-calculator exam.

The entrance exam will cover the following topics

- *Indices*
- *Surds*
- *Factorising*
- *Solving linear and quadratic equations*
- *Changing the subject of a formula*
- *Coordinate Geometry*

This revision pack includes most of the topics with solutions at the back. Please make sure you are fully prepared before sitting the exam.

Indices

Section 1

1. Find:
   (i) 3^4 (ii) 2^6 (iii) $4^{1/2}$
   (iv) 6^0 (v) 5^{-2} (vi) $64^{1/3}$
   (vii) $16^{-1/2}$ (viii) $8^{5/3}$ (ix) $36^{-3/2}$
   (x) $\left(\frac{1}{2}\right)^{-1}$ (xi) $\left(\frac{25}{9}\right)^{-1/2}$ (xii) $\left(\frac{27}{64}\right)^{-2/3}$

2. Simplify the following:
   (i) $3^{11}\times 3^{-4}\div 3^3$ (ii) $\left(2^5\right)^3\times\left(2^7\right)^{-2}$ (iii) $\frac{5^6}{5^5\times 5^3}$

3. Simplify:
   (i) $2^3\times 16^{\frac{1}{2}}$ (ii) $\frac{3^5\times 5^3}{\sqrt{81\times 25}}$

Section 2

1. Simplify the following:
   (i) $\frac{2^5\times 4^{1/2}}{2}$ (ii) $\left(3^5\right)^{3/2}\times 9^{-7/4}$ (iii) $\sqrt{\frac{x^{4/3}}{x^{1/3}\times x^{8/3}}}$

2. Simplify:
   (i) $\frac{16x^{\frac{1}{2}}}{2^3x^{-\frac{1}{2}}}$ (ii) $\frac{x^{\frac{5}{4}}x^{-1}}{\sqrt[4]{x^3}}$

3. Simplify the following:
   (i) $3^{5/2}-3^{1/2}$ (ii) $2^{1/2}+2^{3/2}+2^{5/2}$ (iii) $y^{1/2}-y^{-1/2}$

4. Simplify:
   (i) $\frac{2^{\frac{5}{2}}-2^{\frac{3}{2}}}{2^{\frac{1}{2}}}$
   (ii) $\left(\frac{x^{\frac{7}{4}}-x^{\frac{3}{4}}+x\times x^{\frac{7}{4}}}{x^{\frac{1}{4}}}\right)^2$
   (iii) $\left[\frac{y^{\frac{1}{2}}}{x^{\frac{3}{4}}}-\frac{x^{\frac{5}{4}}}{y^{\frac{3}{2}}}\right]^4$

Surds

1. Write these in terms of the simplest possible surd.

(i) $\sqrt{8}$ (ii) $\sqrt{50}$ (iii) $\sqrt{48}$

(iv) $\sqrt{216}$ (v) $\sqrt{63}$ (vi) $\sqrt{300}$

2. Simplify the following

(i) $\left(1+\sqrt{2}\right)+\left(3-2\sqrt{2}\right)$ (ii) $\left(5\sqrt{2}-2\sqrt{3}\right)-\left(\sqrt{2}+3\sqrt{3}\right)$

(iii) $2\left(\sqrt{5}-3\sqrt{3}\right)+3\left(2\sqrt{5}+\sqrt{3}\right)$ (iv) $\sqrt{18}+\sqrt{72}-\sqrt{98}$

3. Multiply out the brackets and simplify as far as possible.

(i) $\left(1+\sqrt{2}\right)\left(3-\sqrt{2}\right)$ (ii) $\left(2-\sqrt{3}\right)\left(3+2\sqrt{3}\right)$

(iii) $\left(3-2\sqrt{5}\right)\left(1-3\sqrt{5}\right)$ (iv) $\left(3-\sqrt{2}\right)^2$

4. Rationalise the denominators of the following.

(i) $\dfrac{3}{\sqrt{3}}$ (ii) $\dfrac{1}{\sqrt{5}}$

(iii) $\dfrac{1+\sqrt{2}}{\sqrt{2}}$ (iv) $\dfrac{1}{\sqrt{3}+1}$

(v) $\dfrac{\sqrt{2}}{2-\sqrt{2}}$

Quadratic equations

1. Factorise these quadratic expressions.
   (i) x^2+5x+6 (ii) x^2+x-12 (iii) x^2-9
   (iv) x^2-6x+8 (v) $2x^2+3x+1$ (vi) $3x^2+x-2$
   (vii) $4x^2-8x+3$ (viii) $4x^2-25$ (ix) $6x^2-x-12$

2. Factorise:
   (i) x^2-4x (ii) $x^2-17x-60$
   (iii) $x^2+4(x+1)$ (iv) $3x^2-11x+6$

3. Solve these quadratic equations by factorising.
   (i) $x^2+4x+3=0$ (ii) $x^2+5x-6=0$
   (iii) $x^2-6x+8=0$ (iv) $x^2-7x-18=0$
   (v) $2x^2+5x+3=0$ (vi) $2x^2+x-6=0$

4. Write down the equation of the line of symmetry and the coordinates of the vertex of each of the following quadratic graphs:
   (i) $y=(x-4)^2+1$ (ii) $y=(x+2)^2-3$
   (iii) $y=(2x-1)^2-5$ (iv) $y=3-(x+1)^2$

5. A quadratic graph has minimum point (-1, 2). Find an equation for the graph.

6. A quadratic graph has maximum point (2, 5). Find an equation for the graph.

7. Write each of the following quadratic functions in completed square form:
   (i) x^2+2x-3 (ii) x^2-6x+1
   (iii) x^2+x+1 (iv) $-x^2+5x$
   (v) $2x^2+4x+3$ (vi) $3x^2+8x-2$

8. Using your answers for each of the quadratic functions in question 7, write down the coordinates of the minimum or maximum point (the vertex) of the graph.
   (i) $y=x^2+2x-3$ (ii) $y=x^2-6x+1$
   (iii) $y=x^2+x+1$ (iv) $y=-x^2+5x$
   (v) $y=2x^2+4x+3$ (vi) $y=3x^2+8x-2$

Simultaneous equations

1. Solve the following simultaneous equations:

(i) $2x + 5y = 11$
$2x - y = 5$

(ii) $x + 2y = 6$
$4x + 3y = 4$

(iii) $3a - 2b = 4$
$5a + 4b = 3$

(iv) $2p - 5q = 5$
$3p - 2q = -9$

(v) $5x + 3y = 9$
$y = 3x - 4$

(vi) $3a + 2b = 1$
$9a - 4b = 4$

2. Solve the following simultaneous equations:

(i) $x - y = -1$
$3x + 2y = 7$

(ii) $2x + y = 0$
$x - 3y = 7$

(iii) $y - 5x = -8$
$x + 3y = 0$

(iv) $x = 2y - 1$
$-x + 3y = -1$

(v) $2p - 4q = 14$
$-p + 3q = -5$

(vi) $3u - 2v = -26$
$-u + 6v = 46$

Coordinate geometry

1. Given the points A(3, 1), B(6, y) and C(12, -2) find the value(s) of y for which
   (i) the line AB has gradient 2
   (ii) the distance AB is 5
   (iii) A, B and C are collinear
   (iv) AB is perpendicular to BC
   (v) the lengths AB and BC are equal

2. Find the equations of the following lines.
   (i) parallel to $y = 4x - 1$ and passing through (2, 3)
   (ii) perpendicular to $y = 2x + 7$ and passing through (1, 2)
   (iii) parallel to $3y + x = 10$ and passing through (4, -1)
   (iv) perpendicular to $3x + 4y = 12$ and passing through (-3, 0)
   (v) parallel to $x + 5y + 8 = 0$ and passing through (-1, -6)

3. Find the equation of the line AB in each of the following cases.
   (i) A(1, 6), B(3, 2)
   (ii) A(8, -1), B(-2, 3)
   (iii) A(-5, 2), B(7, -4)
   (iv) A(-3, -5), B(5, 1)

4. The point E is (2, -1), F is (1, 3), G is (3, 5) and H is (4, 1).
   Show, by calculation that EFGH is a parallelogram.
   Is EFGH also a rhombus? Explain your answer.

5. P is the point (2, 1), Q is (6, 9) and R is (10, 2).
   (i) Sketch the triangle PQR.
   (ii) Prove that triangle PQR is isosceles.
   (iii) Work out the area of triangle ABC.

6. Three points are A (-1, 5), B (1, 0), and C (11, 4).
   (i) Find the gradient of BA.
   (ii) Find the gradient of BC, and show that BA is perpendicular to BC.
   (iii) Find the equation of the line through C, parallel to BA.
   (iv) Find the equation of the line through A, parallel to BC.
   (v) Find the coordinates of point D, the remaining vertex of the rectangle ABCD.

Indices ANSWERS

Section 1

1. (i) $3^4 = 3\times3\times3\times3 = 81$

(ii) $2^6 = 2\times2\times2\times2\times2\times2 = 64$

(iii) $4^{1/2} = \sqrt{4} = 2$

(iv) $6^0 = 1$

(v) $5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

(vi) $64^{1/3} = \sqrt[3]{64} = 4$

(vii) $16^{-1/2} = \dfrac{1}{\sqrt{16}} = \dfrac{1}{4}$

(viii) $8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$

(ix) $36^{-3/2} = \dfrac{1}{\left(\sqrt{36}\right)^3} = \dfrac{1}{6^3} = \dfrac{1}{216}$

(x) $\left(\dfrac{1}{2}\right)^{-1} = \left(2^{-1}\right)^{-1} = 2^1 = 2$

(xi) $\left(\dfrac{25}{9}\right)^{-1/2} = \left(\dfrac{9}{25}\right)^{1/2} = \sqrt{\dfrac{9}{25}} = \dfrac{3}{5}$

(xii) $\left(\dfrac{27}{64}\right)^{-2/3} = \left(\dfrac{64}{27}\right)^{2/3} = \left(\sqrt[3]{\dfrac{64}{27}}\right)^2 = \left(\dfrac{4}{3}\right)^2 = \dfrac{16}{9}$

2. (i) $3^{11}\times3^{-4}\div3^3 = 3^{11-4-3} = 3^4 = 81$

(ii) $\left(2^5\right)^3\times\left(2^7\right)^{-2} = 2^{15}\times2^{-14} = 2^{15-14} = 2^1 = 2$

(iii) $\dfrac{5^6}{5^5\times5^3} = 5^{6-5-3} = 5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

3. (i) $2^3\times16^{\frac{1}{2}} = 2^3\times\left(2^4\right)^{\frac{1}{2}}$

$= 2^3\times2^2$

$= 2^5 \quad (= 32)$

(ii) $\dfrac{3^5\times5^3}{\sqrt{81\times25}} = \dfrac{3^5\times5^3}{\sqrt{3^4\times5^2}}$

$= \dfrac{3^5\times5^3}{3^2\times5}$

$= 3^3\times5^2 \quad (= 675)$

Indices ANSWERS

Section 2

1. (i) $\dfrac{2^5\times 4^{1/2}}{2}=\dfrac{2^5\times(2^2)^{1/2}}{2}=\dfrac{2^5\times 2^1}{2}=2^{5+1-1}=2^5=32$

(ii) $\left(3^5\right)^{3/2}\times 9^{-7/4}=\left(3^5\right)^{3/2}\times\left(3^2\right)^{-7/4}=3^{15/2}\times 3^{-7/2}=3^{\frac{15}{2}-\frac{7}{2}}=3^4=81$

(iii) $\sqrt{\dfrac{x^{4/3}}{x^{1/3}\times x^{8/3}}}=\sqrt{x^{\frac{4}{3}-\frac{1}{3}-\frac{8}{3}}}=\sqrt{x^{-\frac{5}{3}}}=\left(x^{-\frac{5}{3}}\right)^{\frac{1}{2}}=x^{-\frac{5}{6}}$

2. (i) $\dfrac{16x^{\frac{1}{2}}}{2^3x^{-\frac{1}{2}}}=\dfrac{2^4x^{\frac{1}{2}}}{2^3x^{-\frac{1}{2}}}$

$=2x$

(ii) $\dfrac{x^{\frac{5}{4}}.x^{-1}}{\sqrt[4]{x^3}}=\dfrac{x^{\frac{1}{4}}}{x^{\frac{3}{4}}}$

$=x^{-\frac{1}{2}}$

3. (i) $3^{5/2}-3^{1/2}=3^{1/2}\times 3^2-3^{1/2}$

$=3^{1/2}(3^2-1)$

$=\sqrt{3}\times 8$

$=8\sqrt{3}$

(ii) $2^{1/2}+2^{3/2}+2^{5/2}=2^{1/2}+2^{1/2}\times 2^1+2^{1/2}\times 2^2$

$=2^{1/2}(1+2+2^2)$

$=7\sqrt{2}$

(iii) $y^{1/2}-y^{-1/2}=\sqrt{y}-\dfrac{1}{\sqrt{y}}=\dfrac{y-1}{\sqrt{y}}$

4. (i) $\dfrac{2^{\frac{5}{2}}-2^{\frac{3}{2}}}{2^{\frac{1}{2}}}=\dfrac{2^{\frac{3}{2}}(2-1)}{2^{\frac{1}{2}}}$

$=2$

(ii) $\left(\dfrac{x^{\frac{7}{4}}-x^{\frac{3}{4}}+x.x^{\frac{7}{4}}}{x^{\frac{1}{4}}}\right)^2=\left(\dfrac{x^{\frac{3}{4}}\left[x-1+x^2\right]}{x^{\frac{1}{4}}}\right)^2$

$=\left(x^{\frac{1}{2}}\left[x^2+x-1\right]\right)^2$

$=x\left(x^2+x-1\right)^2$

(iii) $\left[\dfrac{y^{\frac{1}{2}}}{x^{\frac{3}{4}}}-\dfrac{x^{\frac{5}{4}}}{y^{\frac{3}{2}}}\right]^4=\left[\dfrac{y^{\frac{1}{2}}.y^{\frac{3}{2}}-x^{\frac{5}{4}}.x^{\frac{3}{4}}}{x^{\frac{3}{4}}.y^{\frac{3}{2}}}\right]^4$

$=\left[\dfrac{y^2-x^2}{x^{\frac{3}{4}}.y^{\frac{3}{2}}}\right]^4$

$=\dfrac{\left(y^2-x^2\right)^4}{x^3.y^6}$

Surds ANSWERS

1. (i) $\sqrt{8}=\sqrt{4\times 2}=\sqrt{4}\times\sqrt{2}=2\sqrt{2}$

(ii) $\sqrt{50}=\sqrt{25\times 2}=\sqrt{25}\times\sqrt{2}=5\sqrt{2}$

(iii) $\sqrt{48}=\sqrt{16\times 3}=\sqrt{16}\times\sqrt{3}=4\sqrt{3}$

(iv) $\sqrt{216}=\sqrt{36\times 6}=\sqrt{36}\times\sqrt{6}=6\sqrt{6}$

(v) $\sqrt{63}=\sqrt{9\times 7}=\sqrt{9}\times\sqrt{7}=3\sqrt{7}$

(vi) $\sqrt{300}=\sqrt{100\times 3}=\sqrt{100}\times\sqrt{3}=10\sqrt{3}$

2. (i) $(1+\sqrt{2})+(3-2\sqrt{2})=1+3+\sqrt{2}-2\sqrt{2}$
$=4-\sqrt{2}$

(ii) $(5\sqrt{2}-2\sqrt{3})-(\sqrt{2}+3\sqrt{3})=5\sqrt{2}-2\sqrt{3}-\sqrt{2}-3\sqrt{3}$
$=4\sqrt{2}-5\sqrt{3}$

(iii) $2(\sqrt{5}-3\sqrt{3})+3(2\sqrt{5}+\sqrt{3})=2\sqrt{5}-6\sqrt{3}+6\sqrt{5}+3\sqrt{3}$
$=8\sqrt{5}-3\sqrt{3}$

(iv) $\sqrt{18}+\sqrt{72}-\sqrt{98}=\sqrt{9\times 2}+\sqrt{36\times 2}-\sqrt{49\times 2}$
$=3\sqrt{2}+6\sqrt{2}-7\sqrt{2}$
$=2\sqrt{2}$

3. (i) $(1+\sqrt{2})(3-\sqrt{2})=3-\sqrt{2}+3\sqrt{2}-2$
$=1+2\sqrt{2}$

(ii) $(2-\sqrt{3})(3+2\sqrt{3})=6+4\sqrt{3}-3\sqrt{3}-2\times 3$
$=\sqrt{3}$

(iii) $(3-2\sqrt{5})(1-3\sqrt{5})=3-9\sqrt{5}-2\sqrt{5}+6\times 5$
$=33-11\sqrt{5}$

(iv) $(3-\sqrt{2})^2=(3-\sqrt{2})(3-\sqrt{2})$
$=9-3\sqrt{2}-3\sqrt{2}+2$
$=11-6\sqrt{2}$

4. (i) $\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{3}=\sqrt{3}$

(ii) $\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{5}}{5}$

(iii) $\frac{1+\sqrt{2}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{(1+\sqrt{2})\sqrt{2}}{2}=\frac{\sqrt{2}+2}{2}$

(iv) $\frac{1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{\sqrt{3}-1}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{\sqrt{3}-1}{3-1}=\frac{\sqrt{3}-1}{2}$

(v) $\frac{\sqrt{2}}{2-\sqrt{2}}\times\frac{2+\sqrt{2}}{2+\sqrt{2}}=\frac{\sqrt{2}(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}=\frac{2\sqrt{2}+2}{4-2}=\frac{2\sqrt{2}+2}{2}=\sqrt{2}+1$

Quadratic equations ANSWERS

1. (i) $x^2+5x+6=x^2+3x+2x+6$
$=x(x+3)+2(x+3)$
$=(x+2)(x+3)$

(ii) $x^2+x-12=x^2+4x-3x-12$
$=x(x+4)-3(x+4)$
$=(x-3)(x+4)$

(iii) $x^2-9=(x+3)(x-3)$

(iv) $x^2-6x+8=x^2-2x-4x+8$
$=x(x-2)-4(x-2)$
$=(x-4)(x-2)$

(v) $2x^2+3x+1=2x^2+x+2x+1$
$=x(2x+1)+1(2x+1)$
$=(x+1)(2x+1)$

(vi) $3x^2+x-2=3x^2+3x-2x-2$
$=3x(x+1)-2(x+1)$
$=(3x-2)(x+1)$

(vii) $4x^2-8x+3=4x^2-2x-6x+3$
$=2x(2x-1)-3(2x-1)$
$=(2x-3)(2x-1)$

(viii) $4x^2-25=(2x+5)(2x-5)$

(ix) $6x^2-x-12=6x^2+8x-9x-12$
$=2x(3x+4)-3(3x+4)$
$=(2x-3)(3x+4)$

2. (i) $x^2-4x=x(x-4)$

(ii) $x^2-17x-60=(x-20)(x+3)$

(iii) $x^2+4(x+1)=x^2+4x+4$
$=(x+2)^2$

(iv) $3x^2-11x+6=(3x-2)(x-3)$

3. (i) $x^2+4x+3=0$
$(x+3)(x+1)=0$
$x=-3$ or $x=-1$

(ii) $x^2+5x-6=0$
$(x+6)(x-1)=0$
$x=-6$ or $x=1$

(iii) $x^2-6x+8=0$
$(x-2)(x-4)=0$
$x=2$ or $x=4$

(iv) $x^2-7x-18=0$
$(x-9)(x+2)=0$
$x=9$ or $x=-2$

(v) $2x^2+5x+3=0$
$(2x+3)(x+1)=0$
$x=-\frac{3}{2}$ or $x=-1$

(vi) $2x^2+x-6=0$
$(2x-3)(x+2)=0$
$x=\frac{3}{2}$ or $x=-2$

4. (i) Line of symmetry is x = 4
Vertex(minimum point) is (4, 1)

(ii) Line of symmetry is x = -2
Vertex(minimum point) is (-2, -3)

(iii) Line of symmetry is $x=\frac{1}{2}$
Vertex (minimum point) is $\left(\frac{1}{2},-5\right)$

(iv) Line of symmetry is x = -1
Vertex (maximum point) is (-1, 3).

5. Minimum point is (-1, 2)
Equation of graph is $y=(x+1)^2+2$
$=x^2+2x+1+2$
$=x^2+2x+3$

6. Maximum point is (2, 5)
Equation of graph is $y=5-(x-2)^2$
$=5-(x^2-4x+4)$
$=5-x^2+4x-4$
$=-x^2+4x+1$

7. (i) $x^2+2x-3=(x+1)^2-1^2-3$
$=(x+1)^2-4$

(ii) $x^2-6x+1=(x-3)^2-3^2+1$
$=(x-3)^2-8$

(iii) $x^2+x+1=\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1$
$=\left(x+\frac{1}{2}\right)^2-\frac{1}{4}+1$
$=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

(iv) $-x^2+5x=-\left(x^2-5x\right)$
$=-\left(\left(x-\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2\right)$
$=-\left(x-\frac{5}{2}\right)^2+\frac{25}{4}$

(v) $2x^2+4x+3=2\left(x^2+2x\right)+3$
$=2\left(\left(x+1\right)^2-1^2\right)+3$
$=2\left(x+1\right)^2-2+3$
$=2\left(x+1\right)^2+1$

(vi) $3x^2+8x-2=3\left(x^2+\frac{8}{3}x\right)-2$
$=3\left(\left(x+\frac{4}{3}\right)^2-\left(\frac{4}{3}\right)^2\right)-2$
$=3\left(x+\frac{4}{3}\right)^2-3\times\frac{16}{9}-2$
$=3\left(x+\frac{4}{3}\right)^2-\frac{16}{3}-2$
$=3\left(x+\frac{4}{3}\right)^2-\frac{22}{3}$

8. (i) $(-1,-4)$ minimum

(ii) $(3,-8)$ minimum

(iii) $\left(-\frac{1}{2},\frac{3}{4}\right)$ minimum

(iv) $\left(\frac{5}{2},\frac{25}{4}\right)$ maximum

(v) $(-1,\ 1)$ minimum

(vi) $\left(-\frac{4}{3},-\frac{22}{3}\right)$ minimum

Simultaneous equations ANSWERS

1. (i)

$$2x + 5y = 11 \quad (1)$$
$$2x - y = 5 \quad (2)$$

Subtracting: $6y = 6$

$y = 1$

Substituting into (1): $2x + 5 \times 1 = 11$

$2x = 6$

$x = 3$

The solution is $x = 3$, $y = 1$. Check: $2x + 5y = 2 \times 3 + 5 \times 1 = 11$

$2x - y = 2 \times 3 - 1 = 5$

(ii) $x + 2y = 6 \quad (1) \times 4 \qquad 4x + 8y = 24$

$4x + 3y = 4 \quad (2) \qquad 4x + 3y = 4$

Subtracting: $5y = 20$

$y = 4$

Substituting into (1): $x + 2 \times 4 = 6$

$x = -2$

The solution is $x = -2$, $y = 4$. Check: $x + 2y = -2 + 8 = 6$

$4x + 3y = -8 + 12 = 4$

(iii) $3a - 2b = 4 \quad (1) \times 2 \qquad 6a - 4b = 8$

$5a + 4b = 3 \quad (2) \qquad 5a + 4b = 3$

Adding: $11a \quad = 11$

$a = 1$

Substituting into (1): $3 \times 1 - 2b = 4$

$-2b = 1$

$b = -\frac{1}{2}$

The solution is $a = 1$, $b = -\frac{1}{2}$. Check: $3a - 2b = 3 + 1 = 4$

$5a + 4b = 5 - 2 = 3$

(iv) $2p - 5q = 5 \quad (1) \times 3 \qquad 6p - 15q = 15$

$3p - 2q = -9 \quad (2) \times 2 \qquad 6p - 4q = -18$

Subtracting: $-11q = 33$

$q = -3$

Substituting into (1): $2p - 5 \times -3 = 5$

$2p = -10$

$p = -5$

The solution is $p = -5$, $q = -3$. Check: $2p - 5q = -10 + 15 = 5$

$3p - 2q = -15 + 6 = -9$

(v) $5x + 3y = 9 \quad (1)$

$y = 3x - 4 \quad (2)$

Substituting (2) into (1): $5x + 3(3x - 4) = 9$

$5x + 9x - 12 = 9$

$14x = 21$

$x = \frac{3}{2}$

Substituting into (1): $y = 3 \times \frac{3}{2} - 4 = \frac{9}{2} - 4 = \frac{1}{2}$

The solution is $x = \frac{3}{2}, y = \frac{1}{2}$. Check: $5x + 3y = \frac{15}{2} + \frac{3}{2} = 9$

(vi) $3a + 2b = 1 \quad (1) \times 2 \qquad 6a + 4b = 2$

$9a - 4b = 4 \quad (2) \qquad 9a - 4b = 4$

Adding: $15a \quad = 6$

$a = \frac{2}{5}$

Substituting into (1): $3 \times \frac{2}{5} + 2b = 1$

$2b = 1 - \frac{6}{5} = -\frac{1}{5}$

$b = -\frac{1}{10}$

The solution is $a = \frac{2}{5}$, $b = -\frac{1}{10}$. Check: $3a + 2b = \frac{6}{5} - \frac{1}{5} = 1$

$9a - 4b = \frac{18}{5} + \frac{2}{5} = 4$

Simultaneous equations ANSWERS

2. (i)

$x - y = -1 \quad (A)$

$3x + 2y = 7 \quad (B)$

$(A) \Rightarrow 2x - 2y = -2 \quad (C)$

$(C) + (B) \Rightarrow 5x = 5$

$\Rightarrow x = 1, y = 2$

(ii)

$2x + y = 0 \quad (A)$

$x - 3y = 7 \quad (B)$

$(B) \Rightarrow 2x - 6y = 14 \quad (C)$

$(A) - (C) \Rightarrow 7y = -14$

$\Rightarrow y = -2, x = 1$

(iii)

$y - 5x = -8 \quad (A)$

$x + 3y = 0 \quad (B)$

$(B) \Rightarrow 5x + 15y = 0 \quad (C)$

$(A) + (C) \Rightarrow 16y = -8$

$\Rightarrow y = -\frac{1}{2}, x = \frac{3}{2}$

(iv)

$x = 2y - 1 \quad (A)$

$-x + 3y = -1 \quad (B)$

$(A) + (B) \Rightarrow 3y = 2y - 2 \quad (C)$

$\Rightarrow y = -2, x = -5$

(v)

$2p - 4q = 14 \quad (A)$

$-p + 3q = -5 \quad (B)$

$(B) \Rightarrow -2p + 6q = -10 \quad (C)$

$(A) + (C) \Rightarrow 2q = 4$

$\Rightarrow q = 2, p = 11$

(vi)

$3u - 2v = -26 \quad (A)$

$-u + 6v = 46 \quad (B)$

$(B) \Rightarrow -3u + 18v = 138 \quad (C)$

$(A) + (C) \Rightarrow 16v = 112$

$\Rightarrow v = 7, u = -4$

Coordinate geometry ANSWERS

1. (i) Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{1-y}{3-6} = \frac{1-y}{-3}$

Gradient of AB = 2 $\Rightarrow \frac{1-y}{-3} = 2$

$\Rightarrow 1 - y = -6$

$y = 7$

(ii) Distance AB is 5

$\sqrt{(3-6)^2 + (1-y)^2} = 5$

$9 + (1-y)^2 = 25$

$(1-y)^2 = 16$

$1 - y = \pm 4$

$y = 1 - 4$ or $1 + 4$

$y = -3$ or 5

(iii) If A, B and C are collinear, gradient of AB = gradient of AC.

Gradient of AC $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{1-(-2)}{3-12} = \frac{3}{-9} = -\frac{1}{3}$

From (i), gradient of AB $= \frac{1-y}{-3}$

$\frac{1-y}{-3} = -\frac{1}{3}$

$1 - y = 1$

$y = 0$

(iv) If AB is perpendicular to BC, then grad AB × grad BC = -1

From (i), gradient of AB $= \frac{1-y}{-3}$

Gradient of BC $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{y-(-2)}{6-12} = \frac{y+2}{-6}$

$\frac{1-y}{-3} \times \frac{y+2}{-6} = -1$

$(1-y)(y+2) = -18$

$2 - y - y^2 = -18$

$y^2 + y - 20 = 0$

$(y+5)(y-4) = 0$

$y = -5$ or $y = 4$

(v) Length AB = length BC

$\sqrt{(3-6)^2 + (1-y)^2} = \sqrt{(6-12)^2 + (y-(-2))^2}$

$9 + (1-y)^2 = 36 + (y+2)^2$

$1 - 2y + y^2 = 27 + y^2 + 4y + 4$

$0 = 6y + 30$

$y = -5$

2. (i) Gradient of $y = 4x - 1$ is 4

Gradient of parallel line = 4

Equation of line is $y - 3 = 4(x - 2)$

$y - 3 = 4x - 8$

$y = 4x - 5$

(ii) Gradient of $y = 2x + 7$ is 2

Gradient of perpendicular line is $-\frac{1}{2}$

Equation of line is $y - 2 = -\frac{1}{2}(x - 1)$

$2(y - 2) = -(x - 1)$

$2y - 4 = -x + 1$

$2y + x = 5$

(iii) $3y + x = 10 \Rightarrow y = -\frac{1}{3}x + \frac{10}{3}$

Gradient is $-\frac{1}{3}$

Gradient of parallel line is $-\frac{1}{3}$

Equation of line is $y - (-1) = -\frac{1}{3}(x - 4)$

$3(y + 1) = -(x - 4)$

$3y + 3 = -x + 4$

$3y + x = 1$

(iv) $3x + 4y = 12 \Rightarrow y = -\frac{3}{4}x + 3$

Gradient is $-\frac{3}{4}$

Gradient of perpendicular line is $\frac{4}{3}$

Equation of line is $y - 0 = \frac{4}{3}(x - (-3))$

$3y = 4(x + 3)$

$3y = 4x + 12$

(v) $x + 5y + 8 = 0 \Rightarrow y = -\frac{1}{5}x - \frac{8}{5}$

Gradient is $-\frac{1}{5}$

Gradient of parallel line is $-\frac{1}{5}$

Equation of line is $y - (-6) = -\frac{1}{5}(x - (-1))$

$5(y + 6) = -(x + 1)$

$5y + 30 = -x - 1$

$5y + x + 31 = 0$

Coordinate geometry ANSWERS

3. (i) Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{6-2}{1-3} = \frac{4}{-2} = -2$

Equation of AB is $y - 6 = -2(x-1)$

$y - 6 = -2x + 2$

$y + 2x = 8$

(ii) Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{-1-3}{8-(-2)} = \frac{-4}{10} = -\frac{2}{5}$

Equation of AB is $y - (-1) = -\frac{2}{5}(x-8)$

$5(y+1) = -2(x-8)$

$5y + 5 = -2x + 16$

$5y + 2x = 11$

(iii) Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-(-4)}{-5-7} = \frac{6}{-12} = -\frac{1}{2}$

Equation of AB is $y - 2 = -\frac{1}{2}(x-(-5))$

$2(y-2) = -(x+5)$

$2y - 4 = -x - 5$

$2y + x + 1 = 0$

(iv) Gradient of AB $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{-5-1}{-3-5} = \frac{-6}{-8} = \frac{3}{4}$

Equation of AB is $y - (-5) = \frac{3}{4}(x-(-3))$

$4(y+5) = 3(x+3)$

$4y + 20 = 3x + 9$

$4y = 3x - 11$

4. Gradient of EF $= \frac{3-(-1)}{1-2} = \frac{4}{-1} = -4$

Gradient of FG $= \frac{5-3}{3-1} = \frac{2}{2} = 1$

Gradient of GH $= \frac{1-5}{4-3} = \frac{-4}{1} = -4$

Gradient of EH $= \frac{1-(-1)}{4-2} = \frac{2}{2} = 1$

EF is parallel to GH and FG is parallel to EH
so EFGH is a parallelogram.

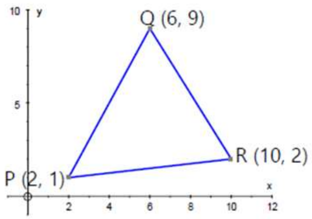
If EFGH were a rhombus, all the sides would be equal.

$EF^2 = (2-1)^2 + (-1-3)^2 = 1^2 + (-4)^2 = 17$

$FG^2 = (1-3)^2 + (3-5)^2 = (-2)^2 + (-2)^2 = 8$

The lengths of EF and FG are not equal, so EFGH is not a rhombus.

5. (i)



(ii) PQ $= \sqrt{(6-2)^2 + (9-1)^2} = \sqrt{16+64} = \sqrt{80}$

PR $= \sqrt{(10-2)^2 + (2-1)^2} = \sqrt{64+1} = \sqrt{65}$

QR $= \sqrt{(10-6)^2 + (2-9)^2} = \sqrt{16+49} = \sqrt{65}$

Since PR and QR are the same length, the triangle is isosceles.

(iii) Take the base of the triangle as PQ.
Let M be the midpoint of PQ

$M = \left(\frac{2+6}{2}, \frac{1+9}{2}\right) = (4,5)$

Height of triangle is MR $= \sqrt{(10-4)^2 + (2-5)^2} = \sqrt{36+9} = \sqrt{45}$

Area of triangle $= \frac{1}{2} \times PQ \times MR$

$= \frac{1}{2}\sqrt{80}\sqrt{45}$

$= \frac{1}{2}\sqrt{16\times5}\sqrt{9\times5}$

$= \frac{1}{2} \times 4\sqrt{5} \times 3\sqrt{5}$

$= 6 \times 5$

$= 30$

6. (i) gradient BA $= \frac{5-0}{(-1)-1} = -\frac{5}{2}$

(ii) gradient BC $= \frac{4-0}{11-1} = \frac{2}{5}$

gradient BA × gradient BC $= -\frac{5}{2} \times \frac{2}{5} = -1$, so BA and BC are perpendicular to each other.

(iii) $y - 4 = -\frac{5}{2}(x-11)$

$\Rightarrow 2y + 5x = 63$

(iv) $y - 5 = \frac{2}{5}(x+1)$

$\Rightarrow 5y - 2x = 27$

(v)
$2y + 5x = 63 \quad (1)$
$5y - 2x = 27 \quad (2)$

mult (1) x 5 $\quad 10y + 25x = 315$

mult (2) x 2 $\quad 10y - 4x = 54$

subtracting $\quad 29x = 261$

$\Rightarrow x = 9, y = 9$

so D is the point (9, 9)