

## Year 12 A-Level Mathematics Entrance Exam Revision Homework

*Please complete all of the questions in this pack, in preparation for your entrance exam.  
DO NOT use a calculator, the exam will be a non-calculator exam.*

The entrance exam will cover the following topics:

- Algebra basics- make sure you are fluent in all the basic algebra skills
- Laws of Indices
- Surds
- Sketching graphs
- Transformations of graphs
- Algebraic fractions
- Changing the subject of a formula
- Solving linear and quadratic equations
- Completing the square
- Solving linear and non-linear simultaneous equations
- Coordinate Geometry – straight lines, circles
- Non right-angled Trigonometry
- Linear and Quadratic Inequalities

*This revision pack includes most of the topics with solutions at the back. Please make sure you are fully prepared before sitting the exam. There will be no opportunities to resit the exam.*

# Algebra basics- make sure you are fluent in all the basic algebra skills

1 Expand.

**a**  $3(2x - 1)$

**c**  $-(3xy - 2y^2)$

2 Expand and simplify.

**a**  $7(3x + 5) + 6(2x - 8)$

**c**  $9(3s + 1) - 5(6s - 10)$

3 Expand.

**a**  $3x(4x + 8)$

**c**  $-2h(6h^2 + 11h - 5)$

4 Expand and simplify.

**a**  $3(y^2 - 8) - 4(y^2 - 5)$

**c**  $4p(2p - 1) - 3p(5p - 2)$

5 Expand  $\frac{1}{2}(2y - 8)$

6 Expand and simplify.

**a**  $13 - 2(m + 7)$

**b**  $-2(5pq + 4q^2)$

**b**  $8(5p - 2) - 3(4p + 9)$

**d**  $2(4x - 3) - (3x + 5)$

**b**  $4k(5k^2 - 12)$

**d**  $-3s(4s^2 - 7s + 2)$

**b**  $2x(x + 5) + 3x(x - 7)$

**d**  $3b(4b - 3) - b(6b - 9)$

**b**  $5p(p^2 + 6p) - 9p(2p - 3)$

## Watch out!

When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '-'.

7 The diagram shows a rectangle.

Write down an expression, in terms of  $x$ , for the area of the rectangle.

Show that the area of the rectangle can be written as  $21x^2 - 35x$

$3x - 5$

$7x$

8 Expand and simplify.

**a**  $(x + 4)(x + 5)$

**c**  $(x + 7)(x - 2)$

**e**  $(2x + 3)(x - 1)$

**g**  $(5x - 3)(2x - 5)$

**i**  $(3x + 4y)(5y + 6x)$

**k**  $(2x - 7)^2$

**b**  $(x + 7)(x + 3)$

**d**  $(x + 5)(x - 5)$

**f**  $(3x - 2)(2x + 1)$

**h**  $(3x - 2)(7 + 4x)$

**j**  $(x + 5)^2$

**l**  $(4x - 3y)^2$

## Extend

9 Expand and simplify  $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

**a**  $\left(x + \frac{1}{x}\right)\left(x - \frac{2}{x}\right)$

**b**  $\left(x + \frac{1}{x}\right)^2$

# Indices

1 Evaluate.

**a**  $14^0$

**b**  $3^0$

**c**  $5^0$

**d**  $x^0$

2 Evaluate.

**a**  $49^{\frac{1}{2}}$

**b**  $64^{\frac{1}{3}}$

**c**  $125^{\frac{1}{3}}$

**d**  $16^{\frac{1}{4}}$

3 Evaluate.

**a**  $25^{\frac{3}{2}}$

**b**  $8^{\frac{5}{3}}$

**c**  $49^{\frac{3}{2}}$

**d**  $16^{\frac{3}{4}}$

4 Evaluate.

**a**  $5^{-2}$

**b**  $4^{-3}$

**c**  $2^{-5}$

**d**  $6^{-2}$

5 Simplify.

**a**  $\frac{3x^2 \times x^3}{2x^2}$

**b**  $\frac{10x^5}{2x^2 \times x}$

**c**  $\frac{3x \times 2x^3}{2x^3}$

**d**  $\frac{7x^3y^2}{14x^5y}$

**e**  $\frac{y^2}{y^{\frac{1}{2}} \times y}$

**f**  $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

**g**  $\frac{(2x^2)^3}{4x^0}$

**h**  $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

**Watch out!**

Remember that any value raised to the power of zero is 1. This is the rule  $a^0 = 1$ .

6 Evaluate.

**a**  $4^{-\frac{1}{2}}$

**b**  $27^{-\frac{2}{3}}$

**c**  $9^{-\frac{1}{2}} \times 2^3$

**d**  $16^{\frac{1}{4}} \times 2^{-3}$

**e**  $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

**f**  $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of  $x$ .

**a**  $\frac{1}{x}$

**b**  $\frac{1}{x^7}$

**c**  $\sqrt[4]{x}$

**d**  $\sqrt[5]{x^2}$

**e**  $\frac{1}{\sqrt[3]{x}}$

**f**  $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

**a**  $x^{-3}$

**b**  $x^0$

**c**  $x^{\frac{1}{5}}$

**d**  $x^{\frac{2}{5}}$

**e**  $x^{-\frac{1}{2}}$

**f**  $x^{-\frac{3}{4}}$

9 Write the following in the form  $ax^n$ .

**a**  $5\sqrt{x}$

**b**  $\frac{2}{x^3}$

**c**  $\frac{1}{3x^4}$

**d**  $\frac{2}{\sqrt{x}}$

**e**  $\frac{4}{\sqrt[3]{x}}$

**f** 3

## Extend

10 Write as sums of powers of  $x$ .

**a**  $\frac{x^5+1}{x^2}$

**b**  $x^2\left(x+\frac{1}{x}\right)$

**c**  $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

# Surds

1 Simplify.

**a**  $\sqrt{45}$

**c**  $\sqrt{48}$

**e**  $\sqrt{300}$

**g**  $\sqrt{72}$

**b**  $\sqrt{125}$

**d**  $\sqrt{175}$

**f**  $\sqrt{28}$

**h**  $\sqrt{162}$

## Hint

One of the two numbers you choose at the start must be a square number.

2 Simplify.

**a**  $\sqrt{72} + \sqrt{162}$

**c**  $\sqrt{50} - \sqrt{8}$

**e**  $2\sqrt{28} + \sqrt{28}$

**b**  $\sqrt{45} - 2\sqrt{5}$

**d**  $\sqrt{75} - \sqrt{48}$

**f**  $2\sqrt{12} - \sqrt{12} + \sqrt{27}$

## Watch out!

Check you have chosen the highest square number at the start.

3 Expand and simplify.

**a**  $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

**c**  $(4 - \sqrt{5})(\sqrt{45} + 2)$

**b**  $(3 + \sqrt{3})(5 - \sqrt{12})$

**d**  $(5 + \sqrt{2})(6 - \sqrt{8})$

4 Rationalise and simplify, if possible.

**a**  $\frac{1}{\sqrt{5}}$

**c**  $\frac{2}{\sqrt{7}}$

**e**  $\frac{2}{\sqrt{2}}$

**g**  $\frac{\sqrt{8}}{\sqrt{24}}$

**b**  $\frac{1}{\sqrt{11}}$

**d**  $\frac{2}{\sqrt{8}}$

**f**  $\frac{5}{\sqrt{5}}$

**h**  $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

**a**  $\frac{1}{3 - \sqrt{5}}$

**b**  $\frac{2}{4 + \sqrt{3}}$

**c**  $\frac{6}{5 - \sqrt{2}}$

## Extend

6 Expand and simplify  $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

7 Rationalise and simplify, if possible.

**a**  $\frac{1}{\sqrt{9} - \sqrt{8}}$

**b**  $\frac{1}{\sqrt{x} - \sqrt{y}}$

# Algebraic Fractions

**1** Factorise.

**a**  $6x^4y^3 - 10x^3y^4$

**c**  $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

**2** Factorise

**a**  $x^2 + 7x + 12$

**c**  $x^2 - 11x + 30$

**e**  $x^2 - 7x - 18$

**g**  $x^2 - 3x - 40$

**b**  $21a^3b^5 + 35a^5b^2$

**b**  $x^2 + 5x - 14$

**d**  $x^2 - 5x - 24$

**f**  $x^2 + x - 20$

**h**  $x^2 + 3x - 28$

**3** Factorise

**a**  $36x^2 - 49y^2$

**c**  $18a^2 - 200b^2c^2$

**b**  $4x^2 - 81y^2$

**4** Factorise

**a**  $2x^2 + x - 3$

**c**  $2x^2 + 7x + 3$

**e**  $10x^2 + 21x + 9$

**b**  $6x^2 + 17x + 5$

**d**  $9x^2 - 15x + 4$

**f**  $12x^2 - 38x + 20$

## Hint

Take the highest common factor outside the bracket.

**5** Simplify the algebraic fractions.

**a**  $\frac{2x^2 + 4x}{x^2 - x}$

**c**  $\frac{x^2 - 2x - 8}{x^2 - 4x}$

**e**  $\frac{x^2 - x - 12}{x^2 - 4x}$

**b**  $\frac{x^2 + 3x}{x^2 + 2x - 3}$

**d**  $\frac{x^2 - 5x}{x^2 - 25}$

**f**  $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

**6** Simplify

**a**  $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

**c**  $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

**b**  $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

**d**  $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

## Extend

**7** Simplify  $\sqrt{x^2 + 10x + 25}$

**8** Simplify  $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

# Completing the square

**1** Write the following quadratic expressions in the form  $(x + p)^2 + q$

**a**  $x^2 + 4x + 3$

**b**  $x^2 - 10x - 3$

**c**  $x^2 - 8x$

**d**  $x^2 + 6x$

**e**  $x^2 - 2x + 7$

**f**  $x^2 + 3x - 2$

**2** Write the following quadratic expressions in the form  $p(x + q)^2 + r$

**a**  $2x^2 - 8x - 16$

**b**  $4x^2 - 8x - 16$

**c**  $3x^2 + 12x - 9$

**d**  $2x^2 + 6x - 8$

**3** Complete the square.

**a**  $2x^2 + 3x + 6$

**b**  $3x^2 - 2x$

**c**  $5x^2 + 3x$

**d**  $3x^2 + 5x + 3$

## Extend

**4** Write  $(25x^2 + 30x + 12)$  in the form  $(ax + b)^2 + c$ .

# Solving quadratic equations

1 Solve

**a**  $6x^2 + 4x = 0$

**c**  $x^2 + 7x + 10 = 0$

**e**  $x^2 - 3x - 4 = 0$

**g**  $x^2 - 10x + 24 = 0$

**i**  $x^2 + 3x - 28 = 0$

**k**  $2x^2 - 7x - 4 = 0$

**b**  $28x^2 - 21x = 0$

**d**  $x^2 - 5x + 6 = 0$

**f**  $x^2 + 3x - 10 = 0$

**h**  $x^2 - 36 = 0$

**j**  $x^2 - 6x + 9 = 0$

**l**  $3x^2 - 13x - 10 = 0$

2 Solve

**a**  $x^2 - 3x = 10$

**c**  $x^2 + 5x = 24$

**e**  $x(x + 2) = 2x + 25$

**g**  $x(3x + 1) = x^2 + 15$

**b**  $x^2 - 3 = 2x$

**d**  $x^2 - 42 = x$

**f**  $x^2 - 30 = 3x - 2$

**h**  $3x(x - 1) = 2(x + 1)$

**Hint**

Get all terms  
onto one side  
of the equation.

3 Solve by completing the square.

**a**  $x^2 - 4x - 3 = 0$

**c**  $x^2 + 8x - 5 = 0$

**e**  $2x^2 + 8x - 5 = 0$

**b**  $x^2 - 10x + 4 = 0$

**d**  $x^2 - 2x - 6 = 0$

**f**  $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

**a**  $(x - 4)(x + 2) = 5$

**b**  $2x^2 + 6x - 7 = 0$

**c**  $x^2 - 5x + 3 = 0$

**Hint**

Get all terms  
onto one side  
of the equation.

5 Solve, giving your solutions in surd form.

**a**  $3x^2 + 6x + 2 = 0$

**b**  $2x^2 - 4x - 7 = 0$

6 Solve the equation  $x^2 - 7x + 2 = 0$

Give your solutions in the form  $\frac{a \pm \sqrt{b}}{c}$ , where  $a$ ,  $b$  and  $c$  are integers.

7 Solve  $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

**Hint**

Get all terms onto one  
side of the equation.

## Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

**a**  $4x(x - 1) = 3x - 2$

**b**  $10 = (x + 1)^2$

**c**  $x(3x - 1) = 10$

# Sketching quadratic graphs

- 1 Sketch the graph of  $y = -x^2$ .
- 2 Sketch each graph, labelling where the curve crosses the axes.  
**a**  $y = (x + 2)(x - 1)$       **b**  $y = x(x - 3)$       **c**  $y = (x + 1)(x + 5)$
- 3 Sketch each graph, labelling where the curve crosses the axes.  
**a**  $y = x^2 - x - 6$       **b**  $y = x^2 - 5x + 4$       **c**  $y = x^2 - 4$   
**d**  $y = x^2 + 4x$       **e**  $y = 9 - x^2$       **f**  $y = x^2 + 2x - 3$
- 4 Sketch the graph of  $y = 2x^2 + 5x - 3$ , labelling where the curve crosses the axes.

## Extend

- 5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.  
**a**  $y = x^2 - 5x + 6$       **b**  $y = -x^2 + 7x - 12$       **c**  $y = -x^2 + 4x$
- 6 Sketch the graph of  $y = x^2 + 2x + 1$ . Label where the curve crosses the axes and write down the equation of the line of symmetry.

# Linear simultaneous equations

Solve these simultaneous equations.

**1**  $4x + y = 8$   
 $x + y = 5$

**2**  $3x + y = 7$   
 $3x + 2y = 5$

**3**  $4x + y = 3$   
 $3x - y = 11$

**4**  $3x + 4y = 7$   
 $x - 4y = 5$

**5**  $2x + y = 11$   
 $x - 3y = 9$

**6**  $2x + 3y = 11$   
 $3x + 2y = 4$

**7**  $y = x - 4$   
 $2x + 5y = 43$

**8**  $y = 2x - 3$   
 $5x - 3y = 11$

**9**  $2y = 4x + 5$   
 $9x + 5y = 22$

**10**  $2x = y - 2$   
 $8x - 5y = -11$

**11**  $3x + 4y = 8$   
 $2x - y = -13$

**12**  $3y = 4x - 7$   
 $2y = 3x - 4$

**13**  $3x = y - 1$   
 $2y - 2x = 3$

**14**  $3x + 2y + 1 = 0$   
 $4y = 8 - x$

## **Extend**

**15** Solve the simultaneous equations  $3x + 5y - 20 = 0$  and  $2(x + y) = \frac{3(y - x)}{4}$ .

# Non-linear simultaneous equations

Solve these simultaneous equations.

**1**  $y = 2x + 1$   
 $x^2 + y^2 = 10$

**2**  $y = 6 - x$   
 $x^2 + y^2 = 20$

**3**  $y = x - 3$   
 $x^2 + y^2 = 5$

**4**  $y = 9 - 2x$   
 $x^2 + y^2 = 17$

**5**  $y = 3x - 5$   
 $y = x^2 - 2x + 1$

**6**  $y = x - 5$   
 $y = x^2 - 5x - 12$

**7**  $y = x + 5$   
 $x^2 + y^2 = 25$

**8**  $y = 2x - 1$   
 $x^2 + xy = 24$

**9**  $y = 2x$   
 $y^2 - xy = 8$

**10**  $2x + y = 11$   
 $xy = 15$

## **Extend**

**11**  $x - y = 1$   
 $x^2 + y^2 = 3$

**12**  $y - x = 2$   
 $x^2 + xy = 3$

# Linear inequalities

**1** Solve these inequalities.

**a**  $4x > 16$

**b**  $5x - 7 \leq 3$

**c**  $1 \geq 3x + 4$

**d**  $5 - 2x < 12$

**e**  $\frac{x}{2} \geq 5$

**f**  $8 < 3 - \frac{x}{3}$

**2** Solve these inequalities.

**a**  $\frac{x}{5} < -4$

**b**  $10 \geq 2x + 3$

**c**  $7 - 3x > -5$

**3** Solve

**a**  $2 - 4x \geq 18$

**b**  $3 \leq 7x + 10 < 45$

**c**  $6 - 2x \geq 4$

**d**  $4x + 17 < 2 - x$

**e**  $4 - 5x < -3x$

**f**  $-4x \geq 24$

**4** Solve these inequalities.

**a**  $3t + 1 < t + 6$

**b**  $2(3n - 1) \geq n + 5$

**5** Solve.

**a**  $3(2 - x) > 2(4 - x) + 4$

**b**  $5(4 - x) > 3(5 - x) + 2$

## **Extend**

**6** Find the set of values of  $x$  for which  $2x + 1 > 11$  and  $4x - 2 > 16 - 2x$ .

# Quadratic inequalities

- 1 Find the set of values of  $x$  for which  $(x + 7)(x - 4) \leq 0$
- 2 Find the set of values of  $x$  for which  $x^2 - 4x - 12 \geq 0$
- 3 Find the set of values of  $x$  for which  $2x^2 - 7x + 3 < 0$
- 4 Find the set of values of  $x$  for which  $4x^2 + 4x - 3 > 0$
- 5 Find the set of values of  $x$  for which  $12 + x - x^2 \geq 0$

## **Extend**

Find the set of values which satisfy the following inequalities.

- 6  $x^2 + x \leq 6$
- 7  $x(2x - 9) < -10$
- 8  $6x^2 \geq 15 + x$

# Straight line graphs

1 Find the gradient and the  $y$ -intercept of the following equations.

- a**  $y = 3x + 5$

**c**  $2y = 4x - 3$

**e**  $2x - 3y - 7 = 0$
- b**  $y = -\frac{1}{2}x - 7$

**d**  $x + y = 5$

**f**  $5x + y - 4 = 0$

**Hint**  
Rearrange the equations  
to the form  $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form  $y = mx + c$ .

| Gradient | $y$ -intercept | Equation of the line |
|----------|----------------|----------------------|
| 5        | 0              |                      |
| -3       | 2              |                      |
| 4        | -7             |                      |

3 Find, in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are integers, an equation for each of the lines with the following gradients and  $y$ -intercepts.

- a** gradient  $-\frac{1}{2}$ ,  $y$ -intercept  $-7$

**c** gradient  $\frac{2}{3}$ ,  $y$ -intercept  $4$
- b** gradient  $2$ ,  $y$ -intercept  $0$

**d** gradient  $-1.2$ ,  $y$ -intercept  $-2$

4 Write an equation for the line which passes through the point  $(2, 5)$  and has gradient  $4$ .

5 Write an equation for the line which passes through the point  $(6, 3)$  and has gradient  $-\frac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.

- a**  $(4, 5), (10, 17)$

**c**  $(-1, -7), (5, 23)$
- b**  $(0, 6), (-4, 8)$

**d**  $(3, 10), (4, 7)$

## Extend

7 The equation of a line is  $2y + 3x - 6 = 0$ .  
Write as much information as possible about this line.

# Parallel and perpendicular lines

- 1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

**a**  $y = 3x + 1$  (3, 2)

**b**  $y = 3 - 2x$  (1, 3)

**c**  $2x + 4y + 3 = 0$  (6, -3)

**d**  $2y - 3x + 2 = 0$  (8, 20)

- 2 Find the equation of the line perpendicular to  $y = \frac{1}{2}x - 3$  which passes through the point (-5, 3).

**Hint**

If  $m = \frac{a}{b}$  then the negative reciprocal  $-\frac{1}{m} = -\frac{b}{a}$

- 3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

**a**  $y = 2x - 6$  (4, 0)

**b**  $y = -\frac{1}{3}x + \frac{1}{2}$  (2, 13)

**c**  $x - 4y - 4 = 0$  (5, 15)

**d**  $5y + 2x - 5 = 0$  (6, 7)

- 4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

**a** (4, 3), (-2, -9)

**b** (0, 3), (-10, 8)

## Extend

- 5 Work out whether these pairs of lines are parallel, perpendicular or neither.

**a**  $y = 2x + 3$   
 $y = 2x - 7$

**b**  $y = 3x$   
 $2x + y - 3 = 0$

**c**  $y = 4x - 3$   
 $4y + x = 2$

**d**  $3x - y + 5 = 0$   
 $x + 3y = 1$

**e**  $2x + 5y - 1 = 0$   
 $y = 2x + 7$

**f**  $2x - y = 6$   
 $6x - 3y + 3 = 0$

- 6 The straight line  $L_1$  passes through the points  $A$  and  $B$  with coordinates (-4, 4) and (2, 1), respectively.

- a** Find the equation of  $L_1$  in the form  $ax + by + c = 0$

The line  $L_2$  is parallel to the line  $L_1$  and passes through the point  $C$  with coordinates (-8, 3).

- b** Find the equation of  $L_2$  in the form  $ax + by + c = 0$

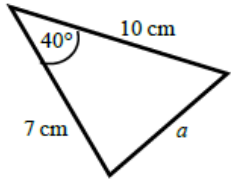
The line  $L_3$  is perpendicular to the line  $L_1$  and passes through the origin.

- c** Find an equation of  $L_3$

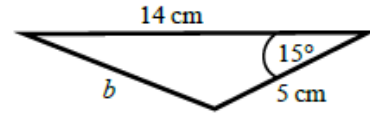
# Non-right angled trigonometry

- 6 Work out the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

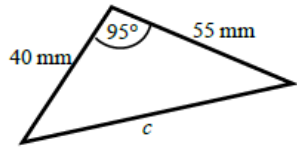
a



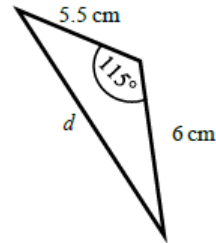
b



c

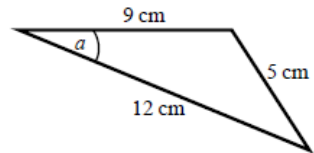


d

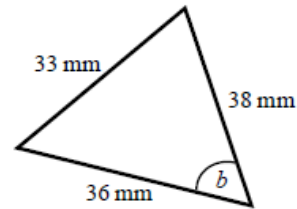


- 7 Calculate the angles labelled  $\theta$  in each triangle. Give your answer correct to 1 decimal place.

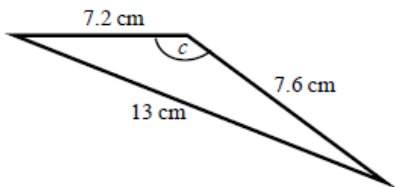
a



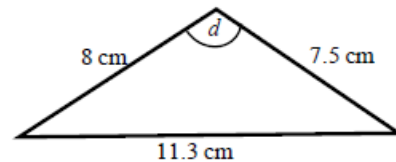
b



c

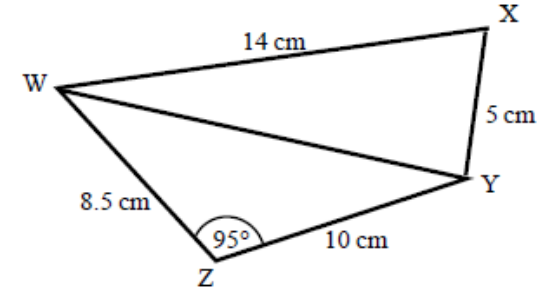


d



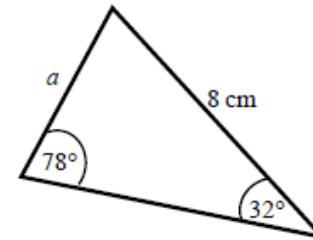
- 8 a Work out the length of WY. Give your answer correct to 3 significant figures.

- b Work out the size of angle WXY. Give your answer correct to 1 decimal place.

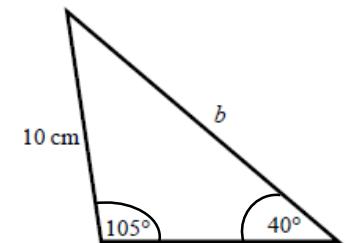


- 9 Find the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

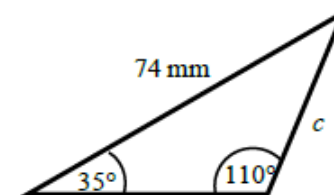
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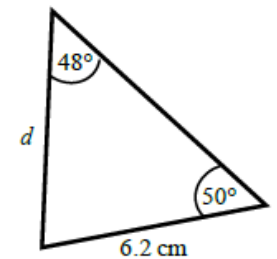
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c

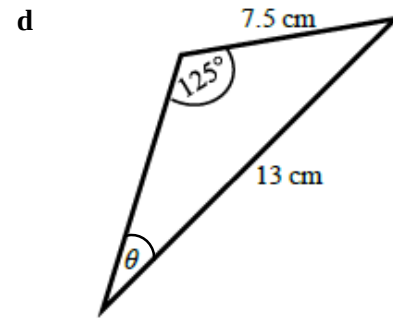
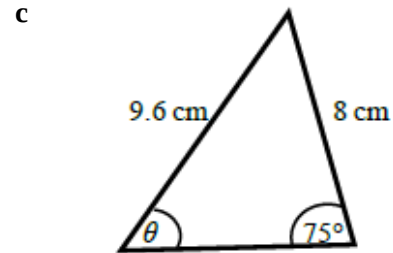
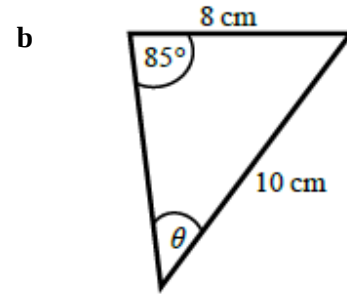
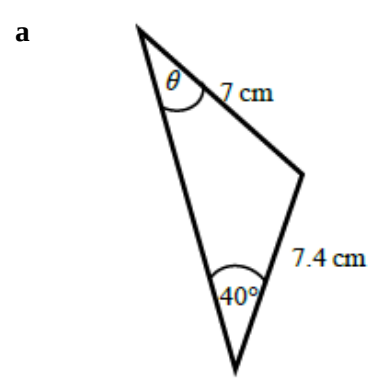


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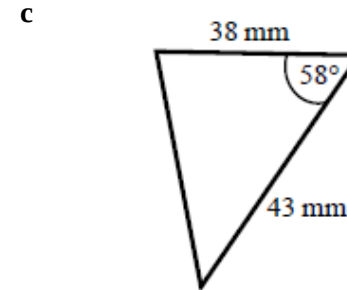
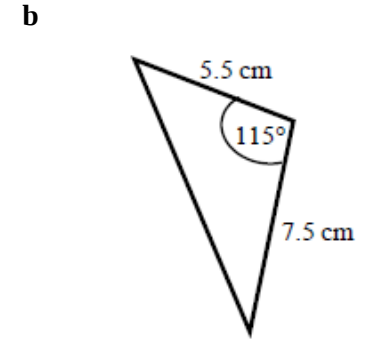
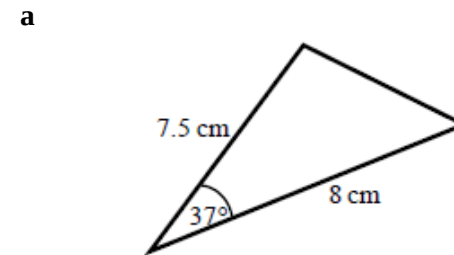


# Non-right angled trigonometry

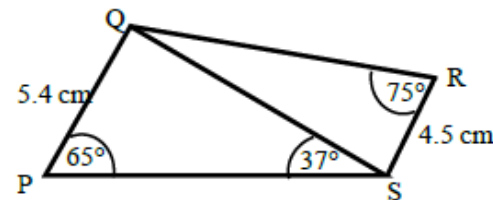
- 10 Calculate the angles labelled  $\theta$  in each triangle.  
Give your answer correct to 1 decimal place.



- 12 Work out the area of each triangle.  
Give your answers correct to 3 significant figures.



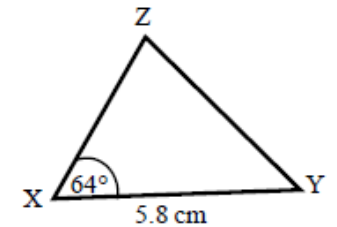
- 11 a Work out the length of QS.  
Give your answer correct to 3 significant figures.
- b Work out the size of angle RQS.  
Give your answer correct to 1 decimal place.



- 13 The area of triangle XYZ is  $13.3 \text{ cm}^2$ .  
Work out the length of XZ.

**Hint:**

Rearrange the formula to make a side the subject.



# Non-right angled trigonometry

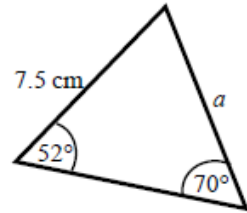
## Extend

- 14 Find the size of each lettered angle or side.  
Give your answers correct to 3 significant figures.

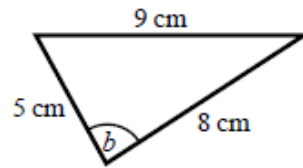
**Hint:**

For each one, decide whether to use the cosine or sine rule.

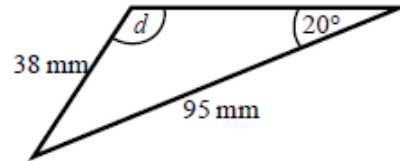
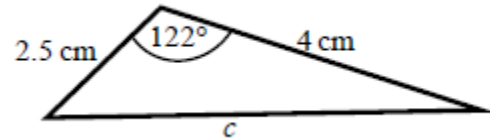
**a**



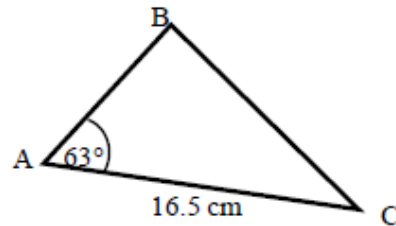
**b**



**c**



- 15 The area of triangle ABC is  $86.7 \text{ cm}^2$ .  
Work out the length of BC.



# Changing the subject of a formula

1  $C = \pi d$  [d]

2  $P = 2l + 2w$  [w]

3  $D = \frac{S}{T}$  [T]

4  $p = \frac{q-r}{t}$  [t]

5  $u = at - \frac{1}{2}t$  [t]

6  $V = ax + 4x$  [x]

7  $\frac{y-7x}{2} = \frac{7-2y}{3}$  [y]

8  $x = \frac{2a-1}{3-a}$  [a]

9  $x = \frac{b-c}{d}$  [d]

10  $h = \frac{7g-9}{2+g}$  [g]

11  $e(9+x) = 2e + 1$  [e]

12  $y = \frac{2x+3}{4-x}$  [x]

13 Make  $r$  the subject of the following formulae.

a  $A = \pi r^2$

b  $V = \frac{4}{3}\pi r^3$

c  $P = \pi r + 2r$

d  $V = \frac{2}{3}\pi r^2 h$

14 Make  $x$  the subject of the following formulae.

a  $\frac{xy}{z} = \frac{ab}{cd}$

b  $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make  $\sin B$  the subject of the formula  $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make  $\cos B$  the subject of the formula  $b^2 = a^2 + c^2 - 2ac \cos B$ .

## Extend

17 Make  $x$  the subject of the following equations.

a  $\frac{p}{q}(sx+t) = x-1$

b  $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$

# ANSWERS

$$\begin{array}{ll} \mathbf{1} & \mathbf{a} \quad 6x - 3 \\ & \mathbf{c} \quad -3xy + 2y^2 \end{array}$$

**b**  $-10pq - 8q^2$

**6 a**  $-1 - 2m$

**b**  $5p^3 + 12p^2 + 27p$

**2 a**  $21x + 35 + 12x - 48 = 33x - 13$

**b**  $40p - 16 - 12p - 27 = 28p - 43$

**c**  $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$

**d**  $8x - 6 - 3x - 5 = 5x - 11$

**3 a**  $12x^2 + 24x$

**b**  $20k^3 - 48k$

**c**  $10h - 12h^3 - 22h^2$

**d**  $21s^2 - 21s^3 - 6s$

**4 a**  $-y^2 - 4$

**b**  $5x^2 - 11x$

$$\mathbf{c} \quad 2p - 7p^2$$

**d**  $6b^2$

**5**  $y - 4$

**7**      $7x(3x - 5) = 21x^2 - 35x$

**8 a**  $x^2 + 9x + 20$

**c**  $x^2 + 5x - 14$

**e**  $2x^2 + x - 3$

**g**  $10x^2 - 31x + 15$

**i**  $18x^2 + 39xy + 20y^2$

**k**  $4x^2 - 28x + 49$

**9**  $2x^2 - 2x + 25$

**10 a**  $x^2 - 1 - \frac{2}{x^2}$

**b**  $x^2 + 10x + 21$

**d**  $x^2 - 25$

**f**  $6x^2 - x - 2$

## h $12x^2 + 13x - 14$

**j**  $x^2 + 10x + 25$

**l**  $16x^2 - 24xy + 9y^2$

**b**  $x^2 + 2 + \frac{1}{x^2}$

## Indices ANSWERS

$$\mathbf{c} \quad x^{\frac{1}{4}}$$

$$\mathbf{f} \quad x^{-\frac{2}{3}}$$

**c**  $\sqrt[5]{x}$

$$\mathbf{f} \quad \frac{1}{\sqrt[4]{x^3}}$$

$$\mathbf{c} \quad \frac{1}{3}x^{-4}$$

**f**  $3x^0$

**c**  $x^{-2} + x^{-7}$

$$\mathbf{f} \quad \frac{16}{9}$$

# Surds ANSWERS

1   **a**    $3\sqrt{5}$   
      **c**    $4\sqrt{3}$   
      **e**    $10\sqrt{3}$   
      **g**    $6\sqrt{2}$

2   **a**    $15\sqrt{2}$   
      **c**    $3\sqrt{2}$   
      **e**    $6\sqrt{7}$

3   **a**    $-1$   
      **c**    $10\sqrt{5}-7$

4   **a**    $\frac{\sqrt{5}}{5}$   
      **c**    $\frac{2\sqrt{7}}{7}$   
      **e**    $\sqrt{2}$   
      **g**    $\frac{\sqrt{3}}{3}$

5   **a**    $\frac{3+\sqrt{5}}{4}$

6    $x-y$

7   **a**    $3+2\sqrt{2}$

**b**    $5\sqrt{5}$   
      **d**    $5\sqrt{7}$   
      **f**    $2\sqrt{7}$   
      **h**    $9\sqrt{2}$

**b**    $\sqrt{5}$   
      **d**    $\sqrt{3}$   
      **f**    $5\sqrt{3}$

**b**    $9-\sqrt{3}$   
      **d**    $26-4\sqrt{2}$

**b**    $\frac{\sqrt{11}}{11}$   
      **d**    $\frac{\sqrt{2}}{2}$   
      **f**    $\sqrt{5}$   
      **h**    $\frac{1}{3}$

**b**    $\frac{2(4-\sqrt{3})}{13}$                       **c**    $\frac{6(5+\sqrt{2})}{23}$

**b**    $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

# Algebraic Fractions ANSWERS

**1**   **a**    $2x^3y^3(3x - 5y)$   
      **c**    $5x^2y^2(5 - 2x + 3y)$

**b**    $7a^3b^2(3b^3 + 5a^2)$

**2**   **a**    $(x + 3)(x + 4)$   
      **c**    $(x - 5)(x - 6)$   
      **e**    $(x - 9)(x + 2)$   
      **g**    $(x - 8)(x + 5)$

**b**    $(x + 7)(x - 2)$   
**d**    $(x - 8)(x + 3)$   
**f**    $(x + 5)(x - 4)$   
**h**    $(x + 7)(x - 4)$

**3**   **a**    $(6x - 7y)(6x + 7y)$   
      **c**    $2(3a - 10bc)(3a + 10bc)$

**b**    $(2x - 9y)(2x + 9y)$

**4**   **a**    $(x - 1)(2x + 3)$   
      **c**    $(2x + 1)(x + 3)$   
      **e**    $(5x + 3)(2x + 3)$

**b**    $(3x + 1)(2x + 5)$   
**d**    $(3x - 1)(3x - 4)$   
**f**    $2(3x - 2)(2x - 5)$

**5**   **a**    $\frac{2(x+2)}{x-1}$

**b**    $\frac{x}{x-1}$

**c**    $\frac{x+2}{x}$

**d**    $\frac{x}{x+5}$

**e**    $\frac{x+3}{x}$

**f**    $\frac{x}{x-5}$

**6**   **a**    $\frac{3x+4}{x+7}$

**b**    $\frac{2x+3}{3x-2}$

**c**    $\frac{2-5x}{2x-3}$

**d**    $\frac{3x+1}{x+4}$

**7**    $(x + 5)$

**8**    $\frac{4(x+2)}{x-2}$

# Completing the square ANSWERS

**1 a**  $(x + 2)^2 - 1$

**b**  $(x - 5)^2 - 28$

**c**  $(x - 4)^2 - 16$

**d**  $(x + 3)^2 - 9$

**e**  $(x - 1)^2 + 6$

**f**  $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$

**2 a**  $2(x - 2)^2 - 24$

**b**  $4(x - 1)^2 - 20$

**c**  $3(x + 2)^2 - 21$

**d**  $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$

**3 a**  $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$

**b**  $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$

**c**  $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$

**d**  $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$

**4**  $(5x + 3)^2 + 3$

# Solving quadratic equations ANSWERS

**1 a**  $x = 0$  or  $x = -\frac{2}{3}$

**c**  $x = -5$  or  $x = -2$

**e**  $x = -1$  or  $x = 4$

**g**  $x = 4$  or  $x = 6$

**i**  $x = -7$  or  $x = 4$

**k**  $x = -\frac{1}{2}$  or  $x = 4$

**b**  $x = 0$  or  $x = \frac{3}{4}$

**d**  $x = 2$  or  $x = 3$

**f**  $x = -5$  or  $x = 2$

**h**  $x = -6$  or  $x = 6$

**j**  $x = 3$

**l**  $x = -\frac{2}{3}$  or  $x = 5$

**2 a**  $x = -2$  or  $x = 5$

**c**  $x = -8$  or  $x = 3$

**e**  $x = -5$  or  $x = 5$

**g**  $x = -3$  or  $x = 2\frac{1}{2}$

**b**  $x = -1$  or  $x = 3$

**d**  $x = -6$  or  $x = 7$

**f**  $x = -4$  or  $x = 7$

**h**  $x = -\frac{1}{3}$  or  $x = 2$

**3 a**  $x = 2 + \sqrt{7}$  or  $x = 2 - \sqrt{7}$

**c**  $x = -4 + \sqrt{21}$  or  $x = -4 - \sqrt{21}$

**e**  $x = -2 + \sqrt{6.5}$  or  $x = -2 - \sqrt{6.5}$

**b**  $x = 5 + \sqrt{21}$  or  $x = 5 - \sqrt{21}$

**d**  $x = 1 + \sqrt{7}$  or  $x = 1 - \sqrt{7}$

**f**  $x = \frac{-3 + \sqrt{89}}{10}$  or  $x = \frac{-3 - \sqrt{89}}{10}$

**4 a**  $x = 1 + \sqrt{14}$  or  $x = 1 - \sqrt{14}$

**c**  $x = \frac{5 + \sqrt{13}}{2}$  or  $x = \frac{5 - \sqrt{13}}{2}$

**b**  $x = \frac{-3 + \sqrt{23}}{2}$  or  $x = \frac{-3 - \sqrt{23}}{2}$

**5 a**  $x = -1 + \frac{\sqrt{3}}{3}$  or  $x = -1 - \frac{\sqrt{3}}{3}$

**b**  $x = 1 + \frac{3\sqrt{2}}{2}$  or  $x = 1 - \frac{3\sqrt{2}}{2}$

**6**  $x = \frac{7 + \sqrt{41}}{2}$  or  $x = \frac{7 - \sqrt{41}}{2}$

**7**  $x = \frac{-3 + \sqrt{89}}{20}$  or  $x = \frac{-3 - \sqrt{89}}{20}$

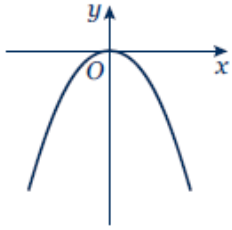
**8 a**  $x = \frac{7 + \sqrt{17}}{8}$  or  $x = \frac{7 - \sqrt{17}}{8}$

**b**  $x = -1 + \sqrt{10}$  or  $x = -1 - \sqrt{10}$

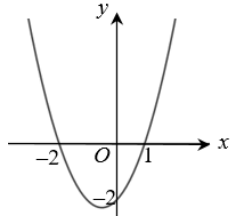
**c**  $x = -1\frac{2}{3}$  or  $x = 2$

# Sketching quadratic graphs ANSWERS

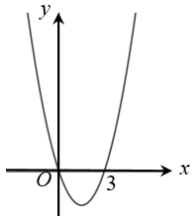
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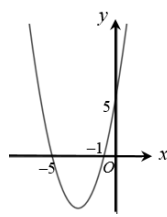
2 a



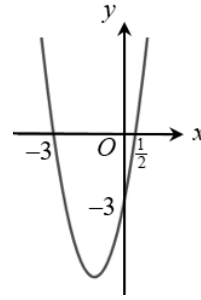
b



c

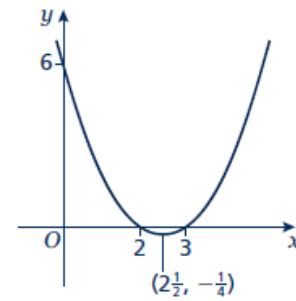


4

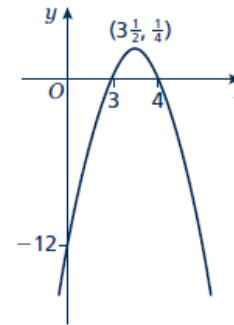


5

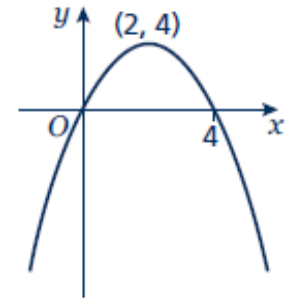
a



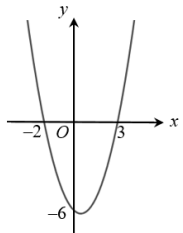
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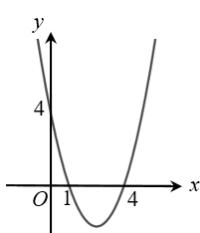
c



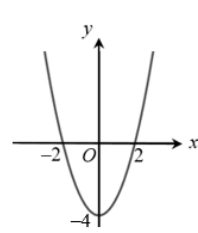
3 a



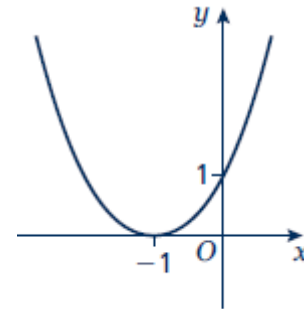
b



c

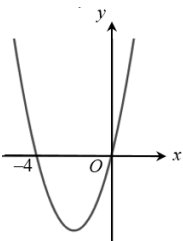


6

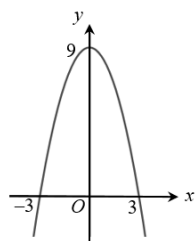


Line of symmetry at  $x = -1$ .

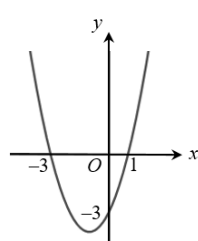
d



e



f



# Linear simultaneous equations ANSWERS

**1**  $x = 1, y = 4$

**2**  $x = 3, y = -2$

**3**  $x = 2, y = -5$

**4**  $x = 3, y = -\frac{1}{2}$

**5**  $x = 6, y = -1$

**6**  $x = -2, y = 5$

**7**  $x = 9, y = 5$

**8**  $x = -2, y = -7$

**9**  $x = \frac{1}{2}, y = 3\frac{1}{2}$

**10**  $x = \frac{1}{2}, y = 3$

**11**  $x = -4, y = 5$

**12**  $x = -2, y = -5$

**13**  $x = \frac{1}{4}, y = 1\frac{3}{4}$

**14**  $x = -2, y = 2\frac{1}{2}$

**15**  $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

# Non-linear simultaneous equations ANSWERS

**1**  $x = 1, y = 3$   
 $x = -\frac{9}{5}, y = -\frac{13}{5}$

**2**  $x = 2, y = 4$   
 $x = 4, y = 2$

**3**  $x = 1, y = -2$   
 $x = 2, y = -1$

**4**  $x = 4, y = 1$   
 $x = \frac{16}{5}, y = \frac{13}{5}$

**5**  $x = 3, y = 4$   
 $x = 2, y = 1$

**6**  $x = 7, y = 2$   
 $x = -1, y = -6$

**7**  $x = 0, y = 5$   
 $x = -5, y = 0$

**8**  $x = -\frac{8}{3}, y = -\frac{19}{3}$   
 $x = 3, y = 5$

**9**  $x = -2, y = -4$   
 $x = 2, y = 4$

**10**  $x = \frac{5}{2}, y = 6$   
 $x = 3, y = 5$

**11**  $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$   
 $x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

**12**  $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$   
 $x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

# Linear inequalities ANSWERS

**1 a**  $x > 4$

**b**  $x \leq 2$

**c**  $x \leq -1$

**d**  $x > -\frac{7}{2}$

**e**  $x \geq 10$

**f**  $x < -15$

**2 a**  $x < -20$

**b**  $x \leq 3.5$

**c**  $x < 4$

**3 a**  $x \leq -4$

**b**  $-1 \leq x < 5$

**c**  $x \leq 1$

**d**  $x < -3$

**e**  $x > 2$

**f**  $x \leq -6$

**4 a**  $t < \frac{5}{2}$

**b**  $n \geq \frac{7}{5}$

**5 a**  $x < -6$

**b**  $x < \frac{3}{2}$

**6**  $x > 5$  (which also satisfies  $x > 3$ )

# Quadratic inequalities ANSWERS

1  $-7 \leq x \leq 4$

2  $x \leq -2$  or  $x \geq 6$

3  $\frac{1}{2} < x < 3$

4  $x < -\frac{3}{2}$  or  $x > \frac{1}{2}$

5  $-3 \leq x \leq 4$

6  $-3 \leq x \leq 2$

7  $2 < x < 2\frac{1}{2}$

8  $x \leq -\frac{3}{2}$  or  $x \geq \frac{5}{3}$

# Straight line graphs ANSWERS

- 1   **a**    $m = 3, c = 5$                       **b**    $m = -\frac{1}{2}, c = -7$   
     **c**    $m = 2, c = -\frac{3}{2}$                       **d**    $m = -1, c = 5$   
     **e**    $m = \frac{2}{3}, c = -\frac{7}{3}$  or  $-2\frac{1}{3}$                       **f**    $m = -5, c = 4$

2

| Gradient | y-intercept | Equation of the line |
|----------|-------------|----------------------|
| 5        | 0           | $y = 5x$             |
| -3       | 2           | $y = -3x + 2$        |
| 4        | -7          | $y = 4x - 7$         |

- 3   **a**    $x + 2y + 14 = 0$                       **b**    $2x - y = 0$   
     **c**    $2x - 3y + 12 = 0$                       **d**    $6x + 5y + 10 = 0$

4    $y = 4x - 3$

5    $y = -\frac{2}{3}x + 7$

6   **a**    $y = 2x - 3$                       **b**    $y = -\frac{1}{2}x + 6$

**c**    $y = 5x - 2$                       **d**    $y = -3x + 19$

7    $y = -\frac{3}{2}x + 3$ , the gradient is  $-\frac{3}{2}$  and the y-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as  $\left(1, \frac{3}{2}\right)$  or  $(4, -3)$ .

# Parallel and perpendicular lines ANSWERS

**1 a**  $y = 3x - 7$

**b**  $y = -2x + 5$

**c**  $y = -\frac{1}{2}x$

**d**  $y = \frac{3}{2}x + 8$

**2**  $y = -2x - 7$

**3 a**  $y = -\frac{1}{2}x + 2$

**b**  $y = 3x + 7$

**c**  $y = -4x + 35$

**d**  $y = \frac{5}{2}x - 8$

**4 a**  $y = -\frac{1}{2}x$

**b**  $y = 2x$

**5 a** Parallel

**b** Neither

**c** Perpendicular

**d** Perpendicular

**e** Neither

**f** Parallel

**6 a**  $x + 2y - 4 = 0$

**b**  $x + 2y + 2 = 0$

**c**  $y = 2x$

## Non-right angled Trigonometry ANSWERS

**6**   **a**   6.46 cm                      **b**   9.26 cm                      **c**   70.8 mm                      **d**   9.70 cm

**7**   **a**   22.2°                      **b**   52.9°                      **c**   122.9°                      **d**   93.6°

**8**   **a**   13.7 cm                      **b**   76.0°

**9**   **a**   4.33 cm                      **b**   15.0 cm                      **c**   45.2 mm                      **d**   6.39 cm

**10**   **a**   42.8°                      **b**   52.8°                      **c**   53.6°                      **d**   28.2°

**11**   **a**   8.13 cm                      **b**   32.3°

**12**   **a**   18.1 cm<sup>2</sup>                      **b**   18.7 cm<sup>2</sup>                      **c**   693 mm<sup>2</sup>

**13**   5.10 cm

**14**   **a**   6.29 cm                      **b**   84.3°                      **c**   5.73 cm                      **d**   58.8°

**15**   15.3 cm

# Changing the subject of a formula ANSWERS

1  $d = \frac{C}{\pi}$

2  $w = \frac{P - 2l}{2}$

3  $T = \frac{S}{D}$

4  $t = \frac{q - r}{p}$

5  $t = \frac{2u}{2a - 1}$

6  $x = \frac{V}{a + 4}$

7  $y = 2 + 3x$

8  $a = \frac{3x + 1}{x + 2}$

9  $d = \frac{b - c}{x}$

10  $g = \frac{2h + 9}{7 - h}$

11  $e = \frac{1}{x + 7}$

12  $x = \frac{4y - 3}{2 + y}$

13 a  $r = \sqrt{\frac{A}{\pi}}$

b  $r = \sqrt[3]{\frac{3V}{4\pi}}$

c  $r = \frac{P}{\pi + 2}$

d  $r = \sqrt{\frac{3V}{2\pi h}}$

14 a  $x = \frac{abz}{cdy}$

b  $x = \frac{3dz}{4\pi cpy^2}$

15  $\sin B = \frac{b \sin A}{a}$

16  $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

17 a  $x = \frac{q + pt}{q - ps}$

b  $x = \frac{3py + 2pqy}{3p - apq} = \frac{y(3 + 2q)}{3 - aq}$